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RESEARCH ARTICLE

INTRODUCTION TO PLANNER HARMONIC MAPPINGS.

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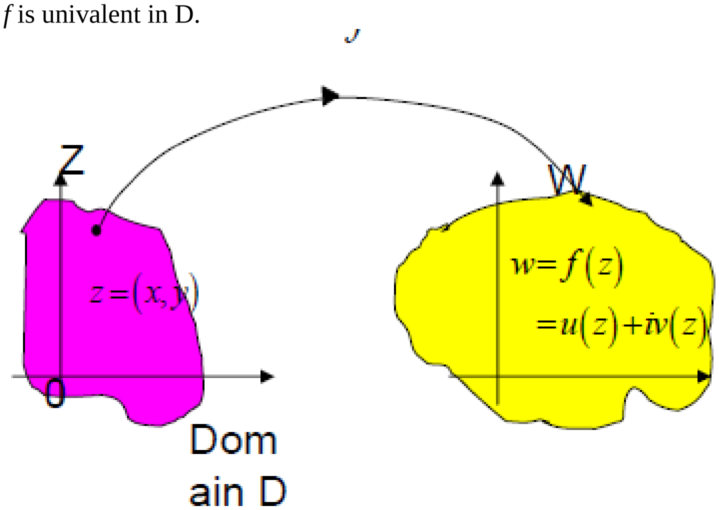
Abstract

In this paper we will study about the planner harmonic mapping starting from basic definition and theorem of harmonic starlike and convex function, harmonic univalent and multivalent function which is very useful for new researchers.

Introduction:-

A complex- valued function $f=u+iv$ in a simply connected domain D is called a **planar harmonic mapping** in complex domain D if the two conditions holds :

(i) u, v are real harmonic in D
(ii) f is univalent in D.



In any simply connected domain D we can write $f=h+\overline{g}$, where h and g are analytic in D and h is called analytic part of f, g is called co-analytic part of f.Clunie and Sheil-Small [15] observed that a necessary and sufficient condition for f to be locally univalent and sense-preserving in D is that $|h'(z)|>|g'(z)|, z\in D$. A family of all harmonic complex-valued, sense-preserving univalent functions normalized with the condition $h(0)=0=h'(0)-1$ is denoted by SH.

SH CLASS:

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Analogous to subclasses of S, various subclasses of SH have been defined and studied.
Note that if the co-analytic part of $f = h + \overline{g}$ is identically zero i.e. $g \equiv 0$ then harmonic functions reduces to analytic functions.

The SH denotes the family of all hamonic, complex valued, orientation-preserving normalized univalent functions defined on Δ. Thus the function f in SH admits the representation $f = h + \overline{g}$, where,

(1) $$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \text{ and } g(z) = \sum_{n=1}^{\infty} b_n z^n;\ |b_1| < 1$$ are analytic functions in Δ.

It follows from the orientation-preserving property that $|b_1| < 1$. Therefore, $(f - \overline{b_1 f})/(1 - |b_1|^2) \in SH$ whenever $f \in SH$. Thus a subclass SH^0 of SH is defined by $SH^0 = \{f \in SH : g'(0) = b_1 = 0\}$.

Note that $S \subset SH^0 \subset SH$. Both families SH and SH^0 are normal families. That is every sequence of functions in SH (or SH^0) has a subsequence that converges locally uniformly in Δ.

It is noted that $SH \equiv S$ if g=0.

TH –CLASS

Let TH denote the sub class of SH with negative coefficients whose members $f = h + \overline{g}$ where h and g are of the form

(2) $$h(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \text{ and } g(z) = \sum_{n=1}^{\infty} |b_n| z^n,\ \ |b_1| < 1,\ z \in \Delta.$$

RESULTS

1.A sense-preserving harmonic mapping $f \in SH$ (or $f \in SH^0$) is said to be harmonic starlike mapping in Δ if the range $f(\Delta)$ is starlike with respect to the origin. Likewise a function $f \in SH(\text{or } f \in SH^0)$ is said to be harmonic convex in Δ if $f(\Delta)$ is a convex domain.

2.A necessary and sufficient condition for a function $f(z) \in SH$ to be harmonic starlike univalent in Δ is that

(3) $$\frac{\partial}{\partial\theta}\{\arg f(re^{i\theta})\} > 0,\ z \in \Delta,\ (z = re^{i\theta},\ 0 \le \theta \le 2\pi,\ 0 \le r \le 1).$$

S^*H-CLASS

S^*H denotes the class of harmonic starlike univalent functions.Note that if $b_1 = 0$ then $S^*H \equiv S^*H^0$.

T^*H denotes the class of harmonic starlike univalent functions $f \in TH$.

4.A necessary and sufficient condition for a function $f(z) \in SH$ to be harmonic convex univalent in Δ is that

(4) $$\frac{\partial}{\partial\theta}\left\{\arg\left(\frac{\partial}{\partial\theta}\arg f(re^{i\theta})\right)\right\} > 0,\ z \in \Delta, (z = re^{i\theta},\ 0 \le \theta \le 2\pi,\ 0 \le r \le 1).$$

KH -CLASS

The class of harmonic convex univalent functions is denoted by KH.

Note that if $b_1 = 0$ then $KH \equiv KH^0$.

T^*KH denotes the class of harmonic convex univalent functions if f is of the form (2).

RESULTS

1.If $f = h + \overline{g}$ with h and g are of the form (1) satisfies

(5) $$\sum_{n=2}^{\infty} n\,|a_n| + \sum_{n=1}^{\infty} n\,|b_n| \le 1$$

then, $f \in S^*H$.

2.If $f = h + \bar{g}$ with h and g are of the form (1.2.4) satisfies

(6) $$\sum_{n=2}^{\infty} n^2\,|a_n| + \sum_{n=1}^{\infty} n^2\,|b_n| \le 1$$

then, $f \in KH$.

3.Let $f = h + \bar{g}$ with h and g are of the form (2), then $f \in T^*H$ if and only if

(7) $$\sum_{n=2}^{\infty} n\,|a_n| + \sum_{n=1}^{\infty} n\,|b_n| \le 1.$$

4.Let $f = h + \bar{g}$ with h and g are of the form (2), then $f \in KH$ if and only if

(8) $$\sum_{n=2}^{\infty} n^2\,|a_n| + \sum_{n=1}^{\infty} n^2\,|b_n| \le 1.$$

RESULTS Of Uniformly starlike and convex

1.A function $f \in SH$ is said to be in the class k-USH(α) if it satisfies the following condition

(9) $$\operatorname{Re}\left\{(1+ke^{i\phi})\frac{zf'(z)}{f(z)} - ke^{i\phi}\right\} \ge \alpha, \quad 0 \le \alpha < 1, \ \phi \in R.$$

2.A function $f \in SH$ given by (1.2.4) is said to be in the class $k - HCV(\alpha)$ if it satisfies the following condition

(10) $$\operatorname{Re}\left\{1 + (1+ke^{i\phi})\frac{z^2h''(z) + \overline{2zg'(z) + z^2g''(z)}}{zh'(z) - \overline{zg'(z)}}\right\} \ge \alpha, 0 \le \alpha < 1, \phi \in R$$

SH (m) -CLASS

Let f be a harmonic function in a Jordan domain D with boundary C. Suppose f is continuous in $\bar{D}$ and $f(z) \ne 0$ on C. Suppose f has no singular zeros in D, and let m to be sum of the orders of the zeros of f in D. Then $\Delta_c \arg(f(z)) = 2\pi m$, where $\Delta_c \arg(f(z))$ denotes the change in argument of f(z) as z traverses C.

It is also shown that if f is sense-preserving harmonic function near a point z_0, where $f(z_0) = \omega_0$ and if $f(z) - \omega_0$ has a zero of order m $(m \ge 1)$ at z_0, then to each sufficiently small $\in > 0$ there corresponds a $\delta > 0$ with the property: "for each $\alpha \in N_\delta(\omega_0) = \{\omega : |\omega - \omega_0| < \delta\}$, the function $f(z) - \alpha$ has exactly m zeros, counted according to multiplicity, in $N_\in(z_0)$". In particular, f has the open mapping property that is, it carries open sets to open sets.

Let Δ be the open unit disc $\Delta = \{z : |z| < 1\}$ also let $a_k = b_k = 0$ for $0 \le k < m$ and $a_m = 1$. Ahuja and Jahangiri [5], [9] introduce and studied certain subclasses of the family SH(m), $m \ge 1$ of all multivalent harmonic and orientation preserving functions in Δ. A function f in SH(m) can be expressed as $f = h + \bar{g}$, where h and g are of the form

(11) $$h(z) = z^m + \sum_{n=2}^{\infty} a_{n+m-1} z^{n+m-1}$$

$$g(z) = \sum_{n=1}^{\infty} b_{n+m-1} z^{n+m-1}, \quad |b_m| < 1.$$

According to Theorem and above argument, functions in SH(m) are harmonic and sense-preserving in Δ if $J_f > 0 \text{ in } \Delta$. The class SH(1) of harmonic univalent functions was studied in details by Clunie and Sheil Small . It was observed that m-valent mapping need not be orientation-preserving.

TH(m) CLASS

Let TH(m) denotes the subclass of SH(m) whose members are of the form

(12) $$h(z) = z^m - \sum_{n=2}^{\infty} |a_{n+m-1}| z^{n+m-1}$$

$$g(z) = \sum_{n=1}^{\infty} |b_{n+m-1}| z^{n+m-1}, \ |b_m| < 1.$$

RESULTS

1. **[5], [9]** A function $f(z) \in SH(m)$ is said to be multivalent harmonic starlike if and only if

(13) $$\frac{\partial}{\partial\theta}\left\{\arg(f(re^{i\theta}))\right\} > 0, \quad z \in \Delta, \ m \geq 1.$$

S^*H_m denotes the class of multivalent harmonic starlike functions

It is clear that $S^*H_m \subset SH(m) \text{ and } S^*H_1 \equiv S^*H$.

Also, T^*H_m denotes the class of harmonic starlike m- valent functions $f \in TH(m)$.

2.[8]A function $f(z) \in SH(m)$ is said to be multivalent harmonic convex if and only if

(14) $$\frac{\partial}{\partial\theta}\left\{\arg\left(\frac{\partial}{\partial\theta} f(re^{i\theta})\right)\right\} > 0, \ z \in \Delta, \ m \geq 1.$$

KH_m denotes the class of multivalent harmonic convex functions.It is clear that $KH_m \subset SH(m)$ and $KH_1 \equiv KH$.

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