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RESEARCH ARTICLE

VISCOUS DISSIPATION AND HEAT TRANSFER EFFECTS IN MHD RAYLEIGH PROBLEM.

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Abstract

In this paper an attempt has been made to study explicitly the effect of viscous dissipation in the MHD Rayleigh problem with inclined magnetic field is investigated in this paper. Numerical results and graphs are presented to study the combined effect of Hartmann number, Hall Parameter, Rotation parameter, Eckert number, angle of inclination and Prandtl number.

Introduction:-

The study of Magnetohydrodynamics of a conducting fluid has applications in a variety of geophysical and astrophysical problems. The unsteady free- convection flow of a electrically conducting, viscous incompressible fluid have gained considerable attention in the presence of applied magnetic field in connection with the theories of fluid motion in the liquid core of the earth, oceanographic and metrological applications.

The *viscosity* of the fluid in a viscous fluid flow will take energy from the motion of the fluid and transform it into internal energy of the fluid. That means heating up the fluid. This partially irreversible process is referred as *dissipation* or *viscous dissipation*. Viscous dissipation is defined as the irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat.

The effect of viscous dissipation in natural convection was analyzed by Gebhart [1]. Iqbal et.al [2] studied the viscous dissipation effects on combined free and forced convection through vertical circular tubes. Hossian [3] studies the effect of viscous and Joules heating on the flow of an electrically conducting fluid past a semiinfinite plate when temperature varies linearly with the distance from the moving edge and it in the presence of a uniform transverse magnetic field. Vajravelu and Hadjiniwlaou [4] analyzed heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. The problem of heat transfer on a moving plate with a uniform magnetic field has attracted the attention of many researchers such as Ali [5], Takharet. al. [6] and Zakaria [7].

The simultaneous effects of the heat transfer and Hall current on a MHD flow with a porous medium in a rotating system was investigated by Dileepsing [8]. Loganathan [9] analyzed the viscous dissipation effects on unsteady natural convective flow past an infinite vertical plate with uniform heat and mass flux. In all the above cases either normal or horizontal magnetic field is considered, but this cannot support the entire physical scenario. In Science

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and Engineering problems oblique magnetic field also play a vital role. Hence this paper mainly deals with the problem involving viscous dissipation and oblique magnetic field.

Author's Contribution:-

The effect of inclined magnetic field with viscous dissipation may become very important in several flow configurations occurring in the Engineering problems. In view of the importance of an inclined magnetic field and dissipation effects, the effect of viscous dissipation in the MHD Rayleigh problem with inclined magnetic field is investigated in this chapter.

Formulation of the Problem:-

An unsteady free convection flow of an electrically conducting viscous incompressible fluid with heat transfer along an infinite flat plate occupying the plane $\mathrm{y} = 0$ has been considered. The x-axis is taken in the direction of the motion of the plate. z – axis lying on the plane normal to both x and y – axis. Initially it is assumed that the plate and the fluid rotate in unison with a uniform angular velocity $\overline{\Omega}$ about the y - axis normal to the plate are at the same temperature Teverywhere in the fluid. At time $t > 0$, the plate starts moving impulsively with the uniform velocity in its own plane along the x-axis. Also the temperature of the plate is raised/lowered to T_∞ and there after maintained uniform. A uniform magnetic field H_0 is applied in a direction which makes an angle θ with the positive direction of y - axis in the $xy -$ plane. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected.

Here $\bar{q}$=$(u,0,w)$ represents the velocity vector, $\bar{H}$=$(H_0\, sin\theta, H_0 cos\theta, 0)$ is the magnetic induction, $\bar{E}$=$(E_x, 0, E_z)$ is the electrostatic field and $\overline{\Omega}$=$(0,\Omega_y,0)$ denotes uniform angular velocity. Under these conditions, the governing boundary layer equations are

Equation of continuity $\nabla.\bar{q}=0$ (1)

Equation of motion $\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{\mathrm{q}} + 2\overline{\Omega} \times \bar{\mathrm{q}} = -\frac{1}{\rho}\nabla\mathrm{P} + \nu\nabla^2\bar{\mathrm{q}} + \frac{1}{\rho}(\bar{\mathrm{J}} \times \bar{\mathrm{B}})$ (2)

Here P is the pressure, ρ is the density, ν is kinematic viscosity and $\bar{\mathrm{J}} \times \bar{\mathrm{B}}$ is the Lorentz force.

The Energy equation $\rho C_P\left[\frac{\partial T}{\partial t} + (\bar{q}.\nabla)T\right] = K\ \nabla^2 T + \mu\Phi$ (3)

whereC_p is the specific heat at constant pressure, K is the thermal conductivity and Φ is the dissipation function due to viscosity.

The generalized Ohm's law is, $\frac{\bar{\mathrm{J}}}{\sigma} = (\bar{\mathrm{E}} + \bar{q} \times \bar{\mathrm{B}}) - \frac{\bar{\mathrm{J}}\times\bar{\mathrm{B}}}{n.e}$ (4)

where$\sigma = \frac{e^2\tau n}{m_e}$ (is the electrical conductivity). Here $\bar{\mathrm{J}}$ is the current density,e is the electric charge, τ is the mean collision time, n is the electron number density and m_e is mass of an electron. As the plate is infinite, all variables in the problem are functions of y and t only. So the term $(\bar{q}.\nabla)\bar{\mathrm{q}}$ reduces to zero and ∇P vanishes as P is constant in equation (2). $T(y,t)$is the temperature of the fluid in the boundary layer and T_∞ is the fluid temperature far away from the plate and let $T(y,t) - T_\infty = \theta_T(y,t)$. Subject to the boundary conditions,

$u = 0,\ w = 0, \theta_T = 0$ for all$t \le 0$ and for all y

$u = U_0, w = 0, \theta_T(0,t) = ae^{i\omega t}$ for all $t > 0$ and $y = 0$

$u \to 0,\ w \to 0, \theta_T \to 0$ for all $t > 0$ and $y \to \infty$ (5)

Let us introduce the following non-dimensional quantities:

$$y^* = \frac{U_0.y}{\nu},\ u^* = \frac{u}{U_0},\ w^* = \frac{w}{U_0},\ t^* = \frac{U_0^2 t}{\nu}, \theta_T{}^* = \frac{\theta_T}{a}, \qquad m = \omega\tau, \qquad M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2},$$

$$Pr = \frac{\nu\rho C_p}{K},\ K^2 = \frac{\nu\Omega_y}{U_0^2}, \xi = \frac{\nu\omega}{U_0^2}, Ec = \frac{U_0^2}{\mathrm{a}C_p} \quad (6)$$

Equations (1), (2) and (3) transform to the following non-dimensional forms (dropping the stars)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2 cos^2\theta}{(1+m^2 cos^2\theta)}(u + mw\ cos\theta) - 2wK^2 \quad (7)$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + \frac{M^2 cos^2\theta}{(1+m^2 cos^2\theta)}(mu\ cos\theta - w) + 2uK^2 \quad (8)$$

$$\frac{\partial \theta_T}{\partial t} = \frac{1}{Pr}\frac{\partial^2\theta_T}{\partial y^2} + Ec\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \quad (9)$$

with the corresponding boundary conditions,

$t \le 0$: $u(y,t) =\ w(y,t) =\ 0, \theta_T(y,0) = 0$ for all y

$t > 0: u(0,t) = 1, w(0,t) = 0, \theta_T(0,t) = e^{i\xi t}$
$t > 0: u(y,t) \to 0,\ w(y,t) \to 0, \theta_T(y,t) \to 0$ as $y \to \infty$ (10)

Solution of the Problem

By using (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \left[\left(\frac{M^2 \cos^2\theta}{1+m^2\cos^2\theta}\right)(1 - im\cos\theta) - 2iK^2\right] q \quad (11)$$

with the boundary conditions,

$$q(y,0) = 0, \quad q(0,t) = 1, \quad q(\infty,t) = 0$$
$$\theta_T(y,0) = 0, \quad \theta_T(0,t) = e^{i\xi t}, \quad \theta_T(\infty,t) = 0 \quad (12)$$

Equation (11) reduces to the form,

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \alpha q \quad (13)$$

where$\alpha = \left(\frac{M^2 \cos^2\theta}{1+m^2\cos^2\theta}\right)(1 - im\cos\theta) - 2iK^2$

$q(y,t) = e^{i\xi t} g(y)$in (13) gives

$$g''(y) - (i\xi + \alpha) g(y) = 0 \quad (14)$$

Equation (14) can be solved under the boundary conditions,

$$g(0) = e^{-i\xi t}, g(\infty) = 0 \quad (15)$$

The solution is

$$g(y) = e^{-i\xi t} e^{-y\sqrt{i\xi + \alpha}} \quad (16)$$

Hence $q(y,t) = e^{i\xi t}\left[e^{-i\xi t} e^{-y\sqrt{i\xi + \alpha}}\right]$ (17)

Real and imaginary parts of equation (17) are

$$u(y,t) = e^{-yS_1} \cos yS_2 \quad (18)$$
$$w(y,t) = -e^{-yS_1} \sin yS_2 \quad (19)$$

where$S_1 = \sqrt{\frac{a+\sqrt{a^2+(\xi+b)^2}}{2}}$ and $S_2 = \sqrt{\frac{-a+\sqrt{a^2+(\xi+b)^2}}{2}}$

Equation (9) reduces to $f''(y) - i\xi \Pr f(y) = \Pr Ec\ e^{-i\xi t} \left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$ (20)

where$\theta_T(y,t) = e^{i\xi t} f(y)$ with the boundary conditions,

$$f(0) = 1, f(\infty) = 0 \quad (21)$$

Hence the solution is

$$\begin{aligned} f(y) = {} & (Cos yS_3 e^{-yS_3} - L_7\, Cos yS_3 e^{-yS_3} - L_8 Sin yS_3 e^{-yS_3} + L_7\, e^{-yS_1}]) \\ & -i(Sin yS_3 e^{-yS_3} - L_8\, Cos yS_3 e^{-yS_3} - L_7 Sin yS_3 e^{-yS_3} - L_8\, e^{-yS_1}]) \end{aligned} \quad (22)$$

where $L_1 = \frac{4S_1^2}{16S_1^4 + \xi^2 Pr^2}$, $L_2 = \frac{\xi Pr}{16S_1^4 + \xi^2 Pr^2}$, $L_3 = \frac{16(S_1^2-S_2^2)^2 - \xi^2 Pr^2 + 64 S_1^2 S_2^2}{(16(S_1^2-S_2^2)^2 - \xi^2 Pr^2 + 64 S_1^2 S_2^2)^2 + 64(S_1^2-S_2^2)\xi^2 Pr^2}$,

$L_4 = \frac{8(S_1^2-S_2^2)\xi Pr}{(16(S_1^2-S_2^2)^2 - \xi^2 Pr^2 + 64 S_1^2 S_2^2)^2 + 64(S_1^2-S_2^2)\xi^2 Pr^2}$,

$L_5 =$
$L_1\, PrEc(S_1^2 + S_2^2) + 8L_3 PrEcS_1\, S_2\, Sin2yS_2\, (S_1^2 - S_2^2) + 2L_4 PrEcS_1\, S_2\, Sin2yS_2\, \xi Pr -$
$16L_3 PrEc(S_1^2 S_2^2) Cos2yS_2$,

$L_6 =$
$L_2\, PrEc(S_1^2 + S_2^2) + 8L_4 PrEcS_1\, S_2\, Sin2yS_2\, (S_1^2 - S_2^2) - 2L_3 PrEcS_1\, S_2\, Sin2yS_2\, \xi Pr -$
$16L_4 PrEc(S_1^2 S_2^2) Cos2yS_2$, $L_7 = L_5\, Cos\xi t + L_6\, Sin\xi t$, $L_8 = L_6\, Cos\xi t - L_5\, Sin\xi t$

Hence

$$\begin{aligned} \theta_T(y,t) = e^{i\xi t}\big(& (Cos yS_3 e^{-yS_3} - L_7\, Cos yS_3 e^{-yS_3} - L_8 Sin yS_3 e^{-yS_3} + L_7\, e^{-yS_1}]) \\ & - i(Sin yS_3 e^{-yS_3} - L_8\, Cos yS_3 e^{-yS_3} - L_7 Sin yS_3 e^{-yS_3} - L_8\, e^{-yS_1}])\big) \end{aligned}$$

Shearing Stress and Nusselt number

The shearing stress at the wall along x - axis and z - axis are given by $\tau_x = \left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $\tau_z = \left(\frac{\partial w}{\partial y}\right)_{y=0}$

From equation (18) and (19), $\tau_x = -S_1$ and $\tau_z = -S_2$

The rate of heat transfer at the plate in terms of the Nusselt number is given by $Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$

Results and Discussion:-

To get a physical insight to the problem graphs are drawn by varying the physical parameters such as Hartmann number, Hall Parameter, Rotation parameter, Eckert number, angle of inclination, Prandtl number and time. Following results are obtained from the graphs.

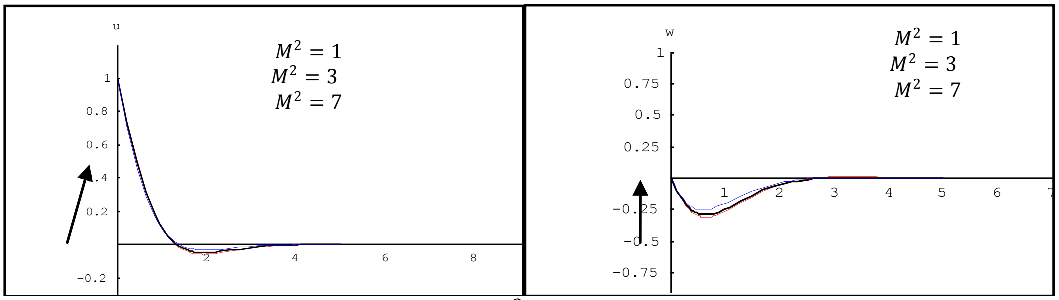

Figure 1:- Effect of Hartmann number (M^2) on primary and secondary velocity profile when$m = 1; K^2 = 2;\ \theta = 30°;\ \xi = 1$

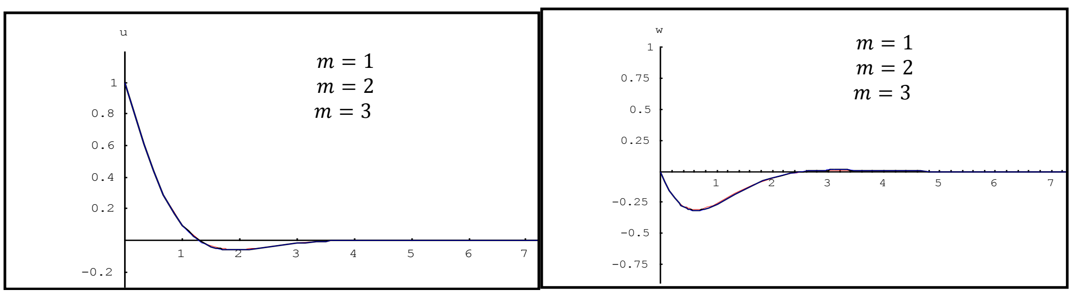

Figure 2:- Effect of Hall parameter (m) on primary and secondary velocity profile when$M^2 = 1; K^2 = 2;\ \theta = 30°;\ \xi = 1$

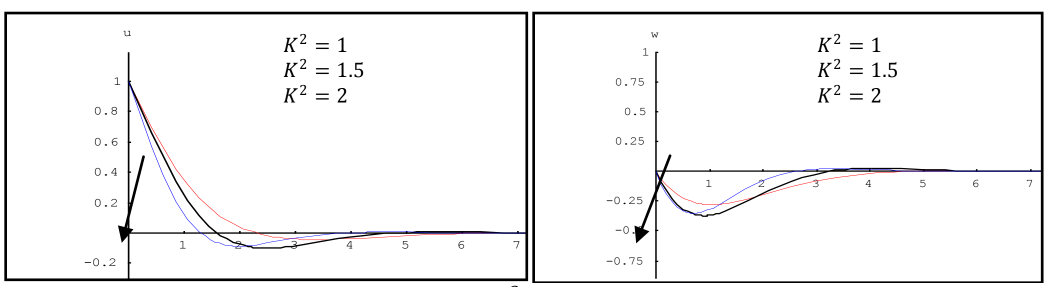

Figure 3:- Effect of Rotation parameter (K^2) on primary and secondary velocity profile when$M^2 = 1; m = 1;\ \theta = 30°;\ \xi = 1$

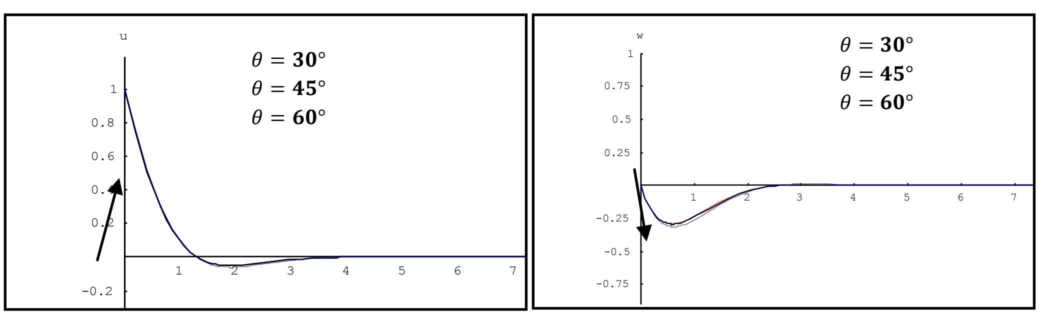

Figure 4:- Effect of angle of inclination (θ) on primary and secondary velocity profile when$M^2 = 1; m = 1; K^2 = 2;\ \xi = 1$

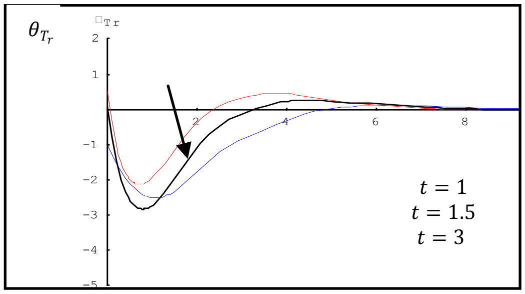

Figure 5:- Temperature profile for various values of time (t)
when $\boldsymbol{Pr = 0.71;\ M^2 = 1;\ m = 1; K^2 = 2;\ \theta = 30°;\ \xi = 1; Ec = 1}$

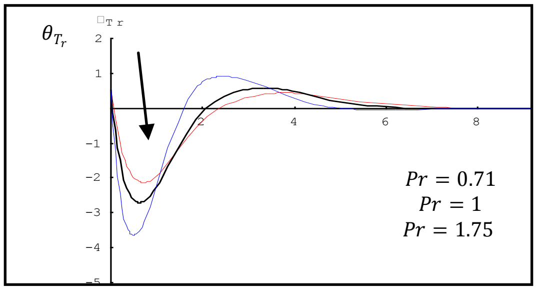

Figure 6:- Temperature profile for various values of Prandtl number (Pr)
when $\boldsymbol{t = 1;\ M^2 = 1;\ m = 1; K^2 = 2;\ \theta = 30°;\ \xi = 1; Ec = 1}$

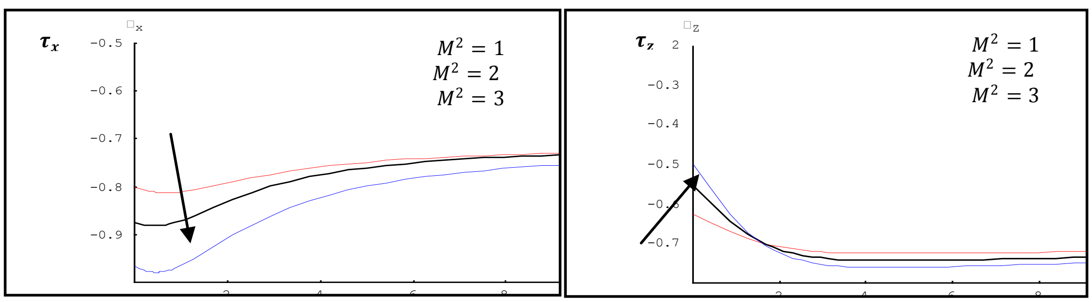

Figure 7:- Shear stress τ_x **and** τ_z **for different** M^2
when $\boldsymbol{K^2 = 2;\ \theta = 30°;\ \xi = 1}$

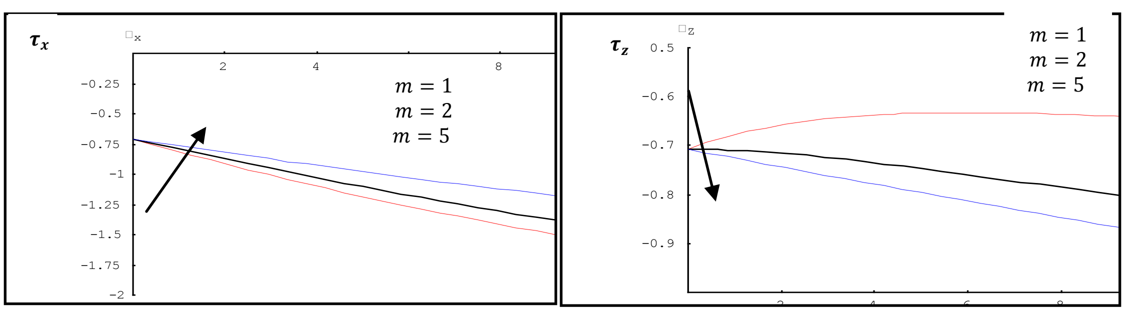

Figure 8 : Shear stress τ_x **and** τ_z **for different** m
when $\boldsymbol{K^2 = 2;\ \theta = 30°;\ \xi = 1}$

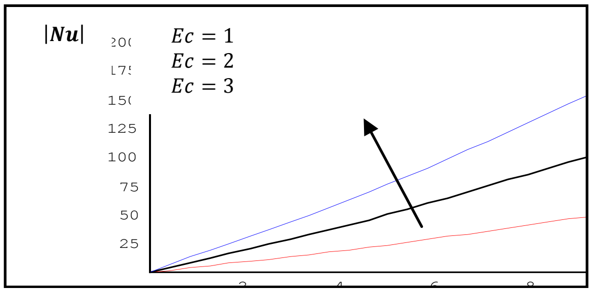

Figure 9:- The amplitude $|Nu|$
when $M^2 = 1$; $m = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $t = 1$

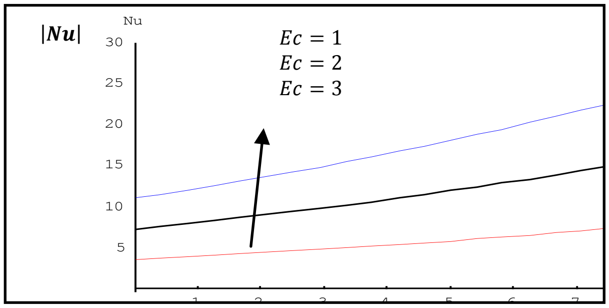

Figure 10:- The amplitude $|Nu|$
when $Pr = 0.71$; $m = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $t = 1$

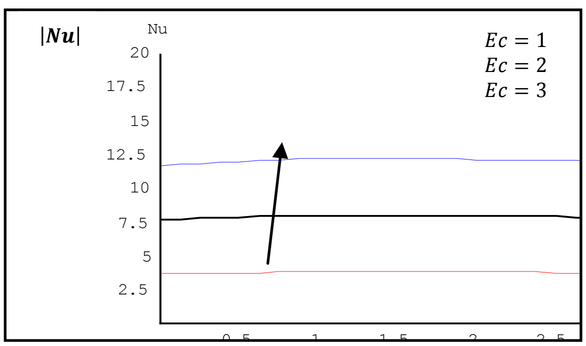

Figure 11:- The amplitude $|Nu|$
when $Pr = 0.71$; $M^2 = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $t = 1$

Figures 1 illustrate the velocity profiles of both primary and secondary for various values of Hartmann number. It is clearly seen from the figure that the profiles increase near the plate and attains a free stream away from the plate. Figure 2 represents the variation of the primary and secondary velocities under the influence of the Hall parameter. It is evident from the figures that when the Hall parameter is increased, both the velocity profiles almost remain as constant. The effect of Rotation parameter retards both the velocity profiles are shown in figures 3. Figure 4 reveals the primary velocity component increase while the secondary velocity component decreases with an increase in the angle of inclination of the magnetic field. This implies that the angle of inclination of the magnetic field accelerates the primary velocity whereas it has a retarding influence in the secondary velocity. In figure 5 and 6, the temperature profile for different values of time and Prandtl number are presented. Here temperature profiles are gradually decreases near the plate and reach the free stream temperature.

The shear stress along the$x$ - axis and z - axis with increase of Hartmann number with respect to the Hall parameter are shown clearly in the figure 7. When the strength of the applied magnetic field is increased, the primary skin friction τ_x decreases due to the Hall effect. The secondary skin friction τ_z increases near the plate. Figure 8 clearly depicts that the shear stress along the x – axis, the primary skin friction has a higher influence and it accelerates whereas the shear stress along the z – axis, the secondary skin friction retards with increase of Hall parameter with respect to Hartmann number. Figures 9 – 11 depicts the profiles of amplitude $|Nu|$ when Eckert number is increased with respect to Prandtl number, Hartmann number and Hall parameter. Eckert number expresses the relation between kinetic energy and the enthalphy. Figures clearly show that the amplitude $|Nu|$ increase considerably when Eckert number increases with the increase of Prandtl number, Hartmann number and Hall parameter.

Conclusion:-

When the magnetic field is increased the velocity profiles also increase but the Hall parameter is not having a notable influence in velocity components. When the rotation parameter is increased the velocity profiles decrease. The primary velocity increase and secondary velocity decrease when the angle of inclination is increased. The temperature profile decreases when time and Prandtl number are increased. The shear stress τ_x at the plate decrease and τ_z at the plate increase with increase of Hartmann number and with increase of Hall parameter τ_x increase and τ_z decrease. With increase of Prandtl number, Hartmann number and Hall parameter, the profile of amplitude $|Nu|$accelerates.

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