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RESEARCH ARTICLE

A THEORETICAL STUDY OF THE ERT INVERSE PROBLEM FOR THE ESTIMATION OF POROSITY IN SUSPENSION

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Abstract

An inverse analysis for an Electrical Resistance Tomography (ERT) was performed to investigate the distribution of the porosity in suspensions. Two Levenberg–Marquardt methods; a standard approach and a modified method, are analyzed and compared to ameliorate the porosity estimation in suspensions by the ERT technique.

Numerical experiments are performed to exemplify the usefulness of the technique. The effects of the measurement errors and the Levenberg Marquardt (LM) coefficient on the stability of the solution were analyzed. Numerical simulations indicate that the modified algorithm is efficient and can overcome the numerical instability of ERT image reconstruction and it improves the quality of the reconstructed images and reduces the computational time compared to LM algorithm.

Introduction:-

The porosity distribution in suspension is essential for understanding many industrial processes, since it has an effect on many rheological properties and subsequently, its behavior in many processes. Most materials physical properties such as density, thermal conductivity and strength are all dependent on their porous structures (Ying Zou et al, 2016; J. Rouquerol et al, 1994; R.W. Rice, 1996; T. Wakuiet al, 2006). For this reason, the particle-volume fraction distribution in suspensions has attracted many researchers interest for a long time (Min Chan K. et al, 2004; Gadala-Maria F. et al, 1980; Leighton, D. et al, 1987; Harit K. et al, 2000).

While the concept of porosity seems raise no difficulty, it is not the same as to its measure. Indeed, this parameter is inaccessible to direct measurement, but there are several methods that measure the essential parameters for its calculation; which are: the void volume accessible to a fluid, the total volume, the volume and density of the solid phase. In many industrial processes, these parameters are not accessible since suspensions are contained in closed tubes or vessels.

Among the direct techniques of porosity measurement: the method of saturation with a fluid under vacuum, and the Purcell method called mercury porosimetry are commonly used. These techniques are extremely useful for porous materials (Antoine I., 1990). Y. Zou et al (2016) compared different porosity measurement methods to ascertain a relatively accurate and efficient method suitable for laboratory utilization. Different indirect methods rely on completely different physical principles, are also used (F.Andreola, 2000; F. Dullien, 2012). For example, Madiouli et al. (2011), (2007) presented graphical interpretation to estimate porosity changes in the material structure. Non destructive methods are also used to determine the porosity. Leonard (2004) used the X-ray microtomography to follow changes of texture of sludge during a drying treatment. In geophysics, the porosity and the hydraulic conductivity were estimated by Vertical Electrical Sounding (VES) (Sri Niwas and Muhammed Celik, 2012).

The electrical tomography technique is one of the non destructive techniques. It is applied in many industrial processes to obtain information about the contents of closed pipes or vessels by measuring variations in the dielectric property of material inside the vessel (T. Dahlin, 1996; T. Dahlin and Bing Zhou, 2004; B. Zhou and T. Dahlin, 2003; Sasaki Y, 1992). In geophysics, this technique is used to represent changes in soil electrical resistivity (T. Dahlin, 2001).

The electrical tomographic methods applied in two-phase flow measurement have become popular (F. Dong et al, 2003; M. Wang et al, 1999; Yixin Ma et al, 2001; G.P. Lucas et al, 1999). In the same field, R. Giguère (2008) has worked on image reconstruction for bi-dimensional ERT to visualize multiphase flow. Rosa Ma**. (**2012) considers 2D ERT as a powerful tool for the diagnosis of the subsoil state.

In this work, we propose to use the ERT for the estimation of porosity distribution in suspensions using boundary voltage measurements. This non-instructive image reconstruction technique consists in injecting currents through an array of electrodes, and measuring voltages at the boundaries of the vessel. If the electric current is carried by the interstitial solution which fills the pores, the obtained image is a representation of the porous structure of the material which will allow the monitoring of the porosity.

The inverse problem for the ERT is solved in order to estimate the electrical conductivity to predict volume fraction and suspension porosity. The majority of parameter estimation problems are ill-posed due to the presence of data noise. Therefore, regularization methods have to be used to obtain stable approximations of the solution (T.Feng, 2007).

In the present work, an iterative regularization method, the Levenberg–Marquardt method (T.Feng, 2007; A. Fguiri, 2007), is applied. Performance of this algorithm has been studied for synthetic test cases. This iterative algorithm, based on the minimization of a cost (objective) function, presents a good exchange between the speed of the Newton algorithm and the stability of the steepest descent method (A.Abolfazl Suratgar, 2007). Its principle disadvantage is that there is no guarantee it will find the global minimum (A. Fguiri, 2007).

At the end to ameliorate the resolution of the ERT inverse problem, two Levenberg–Marquardt methods, a standard approach and a modified method are analyzed and compared.

These two algorithms are tested by the noise-free voltage data and the noise-contaminated voltage data. Numerical simulations indicate that the modified algorithm is efficient and can overcome the numerical instability of ERT image reconstruction and it improves the quality of the reconstructed images and reduces the computational time compared to some other methods. Porosity Estimation results indicate that the convergence of the numerical method depends mainly on some parameters such as the initial value of estimated parameter, initial value of damping parameter and the noise measurement.

Theoretical Method:-

The porosity describes the fraction of void space. It is defined by the ratio:

$$\varepsilon = \frac{V_V}{V_T} = \frac{V_T - V_s}{V_T} = 1 - \frac{V_s}{V_T} \tag{1}$$

Where V_V is the volume of void-space (liquid volume in suspension) and V_T is the total volume of the suspension.

The ratio $\frac{V_s}{V_T}$ is defined as the particles volume fraction:

$$\frac{V_s}{V_T} = C_V \tag{2}$$

The relationship between dimensionless conductivity and the volume fraction of the particles of the suspension is given (Min Chan K. et al, 2004; Meredith R. E. and Tobias C.W, 1961).

$$C_V = \frac{24-(63+448\sigma_d+64\sigma_d^2)^{1/2}}{2(8+\sigma_d)} \quad (3)$$

With $\sigma_d = \frac{\sigma}{\sigma_0}$ is the ratio of the conductivity of the suspension relative to pure liquid.

Using equations (1), (2) and (3) we obtain a relationship between the electrical conductivity and the porosity:

$$\varepsilon = 1 - \frac{24-(63+448\sigma_d+64\sigma_d^2)^{1/2}}{2(8+\sigma_d)}$$

Forward Problem:-

We consider a theoretical study for the estimation of the porosity using an ERT system. In the considered system an array of 10 electrodes is attached on the boundary of an object. This technique consists in injecting small alternating currents through these electrodes and the resulting voltages are measured. From these boundary measurement data the internal conductivity distribution, so, the porosity distribution can be obtained. A schematic diagram of the studied configuration is shown in Fig. 1.

The considered configuration is a rectangle (the vertical plane of the cylinder) where we put electrodes on its two sides, each side contains 5 electrodes. This allows to have an electrical conductivity at a given distance z coast constant over the entire section of the filter. The electrical conductivity was assumed constant in a horizontal plane.

The mathematical model of the ERT is defined by the tomography equation which derives from Maxwell equations

$$\nabla.(\sigma(z)\nabla V(x,z))=0 \quad (4)$$

With V is electrical potential and σ is the electrical conductivity.

Boundary conditions are:

$$\sigma\, \partial V/\partial x = -I/A_e \text{ on the cathode;} \quad (5)$$

$$\sigma \partial V/\partial x = I/A_e \text{ on the anode;} \quad (6)$$

$$\sigma\, \partial V/\partial x = 0 \text{ on insulating surface} \quad (7)$$

Solving the forward problem allows to determine the distribution of electric potential in the suspension by using assumed conductivity distribution.

Inverse Problem:-

In terms of mathematics, the ERT reconstruction problem is a nonlinear inverse problem. It needs a reconstruction algorithm.

For the ERT inverse problem considered, the electrical conductivity distribution is regarded as unknown, while the boundary voltages are calculated using the direct problem described above with high degree of accuracy (M. M. Mejias et al, 1999).

Solving the inverse problem consists in minimizing the difference between the predictions of the numerical voltages and the physical reality represented by the experimental ones. The Levenberg Marquardt method was applied.

The cost function to be minimized is defined as the sum of squared errors between the measured and the simulated voltage, namely:

$$O_b(\sigma) = \left\|U_m - U_{cal}\right\|^2 = (U_m - U_{cal}(\sigma))^T (U_m - U_{cal}(\sigma)) \quad (8)$$

Where U_m is the vector of electrical voltage determined by a simulated experimentation and $U_{cal}(\sigma)$ is the vector of voltage obtained from the solution of the direct problem.

To minimize the cost function, it is necessary to make corrections Δσ such as:

$$O_b(\sigma_0+\Delta\sigma) \prec O_b(\sigma_0)$$

$\sigma 0$ is the initial conductivity of the medium.

A Taylor expansion of 2^{nd} order of the objective function gives

$$O_b(\sigma_0+\Delta\sigma)=\mathrm{O}_b(\sigma_0)+\left[\nabla O_b(\sigma_0)\right]^T.\Delta\sigma+\Delta\sigma^T\left[\nabla^2 O_b(\sigma_0)\right].\Delta\sigma+\delta$$

With

The minimum of the function is reached in $\sigma_0+\Delta\sigma$

Therefore its gradient is equal to zero

$$O_b(\sigma_0+\Delta\sigma)=0$$

$$\nabla O_b(\sigma_0+\Delta\sigma)=\nabla O_b(\sigma_0)+\left[\nabla^2 O_b(\sigma_0)\right]^T.\Delta\sigma=0$$

This implies

$$\Delta\sigma=-\left[\nabla^2 O_b(\sigma_0)\right]^{-1}\nabla O_b(\sigma_0)$$

The iterative minimization of the objective function takes the form:

$$\{\sigma\}^{k+1}=\{\sigma\}^k-([H]^{-1})^k\{g\} \quad (9)$$

With $[H]=\nabla^2 O_b$ Hessian Matrix.

$\{g\}=\nabla O_b$: Gradient of the objective function.

And k is the iteration number.

Marquardt (1963) was thought to improve the convergence of this algorithm, adding a scalar to the diagonal of the Hessian matrix.

The iterative minimization of the objective/cost function becomes:

$$\{\sigma\}^{k+1}=\{\sigma\}^k-([H+\lambda.\Omega]^{-1})^k\{g\} \quad (10)$$

The coefficient λ is a damping parameter added to the diagonal of H= (J^TJ) in order to control the stability of the algorithm. Iterations are generally started with large values of λ. Then this parameter is gradually reduced as the solution approaches the converged result.

The Hessian matrix is given as:
H= (J^TJ)
Where **J** is the jacobian matrix defined as:

$$J(\sigma)=\left[\frac{\partial U_{cal}(\sigma)}{\partial\sigma}\right]=\begin{bmatrix}\frac{\partial U_{cal,1}}{\partial\sigma_1} & \cdots & \frac{\partial U_{cal,1}}{\partial\sigma_N}\\ \vdots & \ddots & \vdots\\ \frac{\partial U_{cal,\mathrm{M}}}{\partial\sigma_1} & \cdots & \frac{\partial U_{cal,M}}{\partial\sigma_N}\end{bmatrix}$$

Where M: the measurement number.
And N: Nodes number with respect to z.

In the standard LM method, λ is defined as a constant number. A. Abolfazl (2007) proposed a modified LM method using:

$$\lambda=0.01(U_m-U_{cal})^T(U_m-U_{cal})$$

The choice of a damping parameter depending on the difference between the measured and the simulated voltage can speed up the LM algorithm. This algorithm was used to solve the inverse problem of the ERT.

Result and Discussion:-

The resolution of ERT problem depends on various variables, such as electrical conductivity distribution, current injection strategy, and errors in current injection and voltage measurement. Therefore, to validate the use of ERT in this problem, a series of numerical simulations should be carried out.

The influence of the LM parameter λ has been also analyzed. We performed the estimation of the porosity by varying λ between 1000 and 0.1. As can be seen in Fig.2, the choice of initial value of this ratio has an influence on the identification. The convergence of LM method is faster with low λ values. This is explained by its tendency to the Gauss-Newton method known for its rapid convergence but a problem of choice of the initial value. When the damping parameter λ is made large, the LM method tends to the Steepest Descent method which does not require a good initial guess but converges slowly.

To test the robustness of the algorithm, we investigate the effect of the measurement errors level on the reconstruction. We propose several artificial conductivity distributions (in other word porosity distributions) and boundary voltage were calculated using the forward problem.

We consider two examples of configuration, the first is for the case where the porosity distribution presents a minimum (Fig.3), and the second is for a linear distribution example (Fig 4).

The porosity spatial evolution is presented for different measurement noise values ζ. It is noted that identification is best for small noise values. With noise measurement level of 0.1 and 1% the estimated porosity is similar to that exact value. This allows us to conclude that our algorithm is not sensitive to low measurement noise values. The estimation error is even higher when the measurement noise is higher.

Besides, according to Fig.4 we note that using a noise level of 40% in the case of a linear distribution, the porosity estimation presents oscillations.

For these two examples of configuration, the error calculated during the estimation of porosity is given in Fig.5 and Fig.6. The error is defined as the difference between the estimated and the exact value of porosity.

Using a noise level ζ = 40%, the error presents a gradient between different heights z, it oscillates between 28 and 37% for the first example. This gradient decreases gradually as the measurement noise decreases until zero for $\zeta \leq$ 1%.

In this section we consider a comparison between the standard LM and modified algorithm applied to solve the inverse problem of the ERT. Simulated porosity, conductivity and cost function are investigated with standard LM and modified LM.

We consider an initial guess of the conductivity quite far from the converged value ($\sigma 0$ =30 S/m witch correspond to initial porosity $\varepsilon 0$= 4,514%). The identification of porosity and conductivity during iterations using standard LM and modified LM method is examined in Fig.7 and Fig.8.

As can be seen, the modified algorithm converges to the value of the exact/converged porosity (ε= 3,138%) .However, the standard LM method overestimates the converged result. The final estimated porosity is 3,349% while the exact value is 3,138. This is explained by the tendency of the LM method to that of Gauss-Newton which converges fast but requires a good initial guess. It is the same for the conductivity (Fig.8), accuracy of estimates is significantly improved when using modified damping parameter since it converges to 5S/m instead of 6,11 S/m with standard method.

According to Fig.9, the cost function presents a fast convergence using the modified algorithm. For that, we can conclude that using the modified LM algorithm in solving the inverse problem of ERT ameliorate the computational efficiency and the rate of convergence even with an initial value far from the converged value.

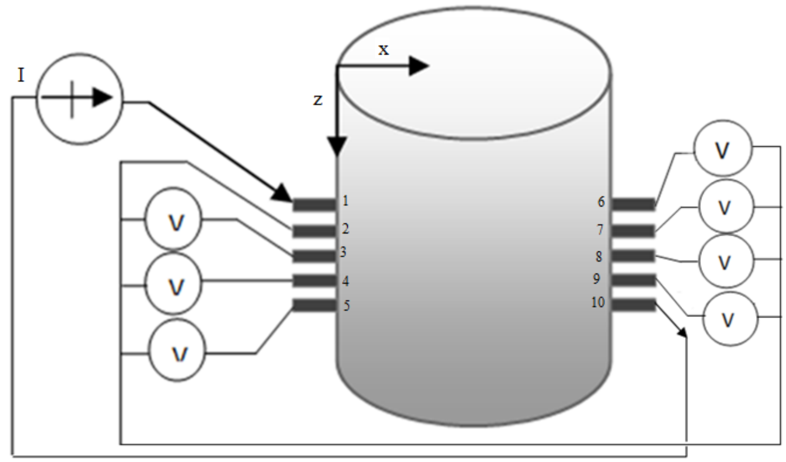

Fig.1. Electrodes model

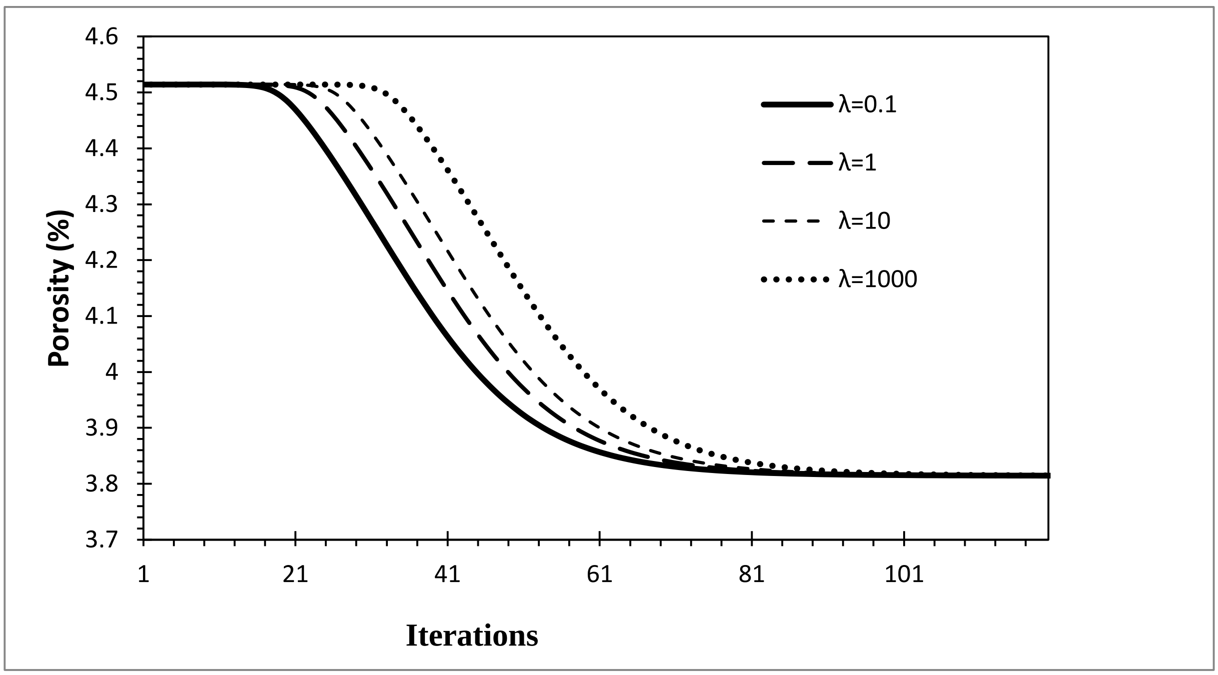

Fig.2. Influence of the LM coefficient on the porosity estimation

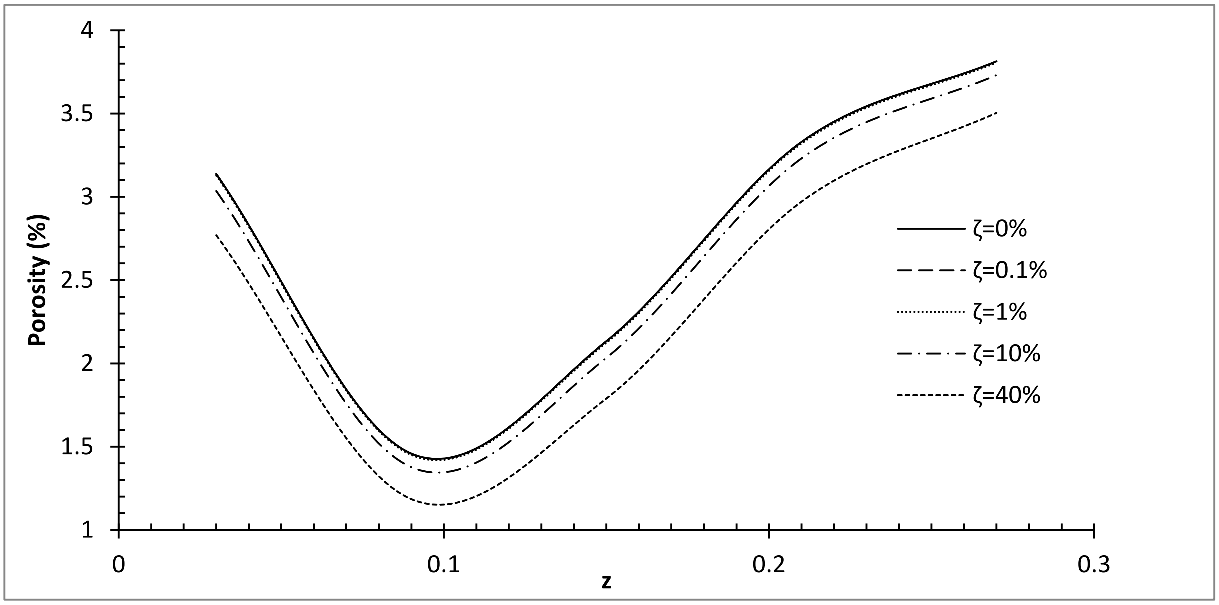

Fig.3. Computer simulation result for porosity distribution showing a minimum

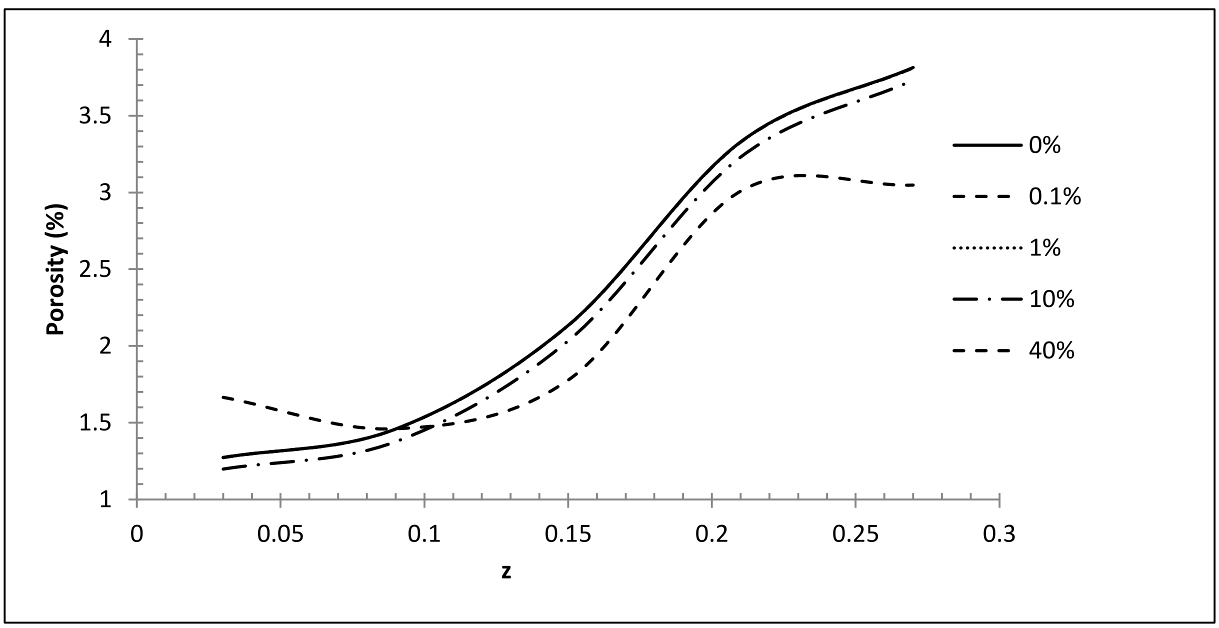

Fig.4. Computer simulation result for linear porosity distribution

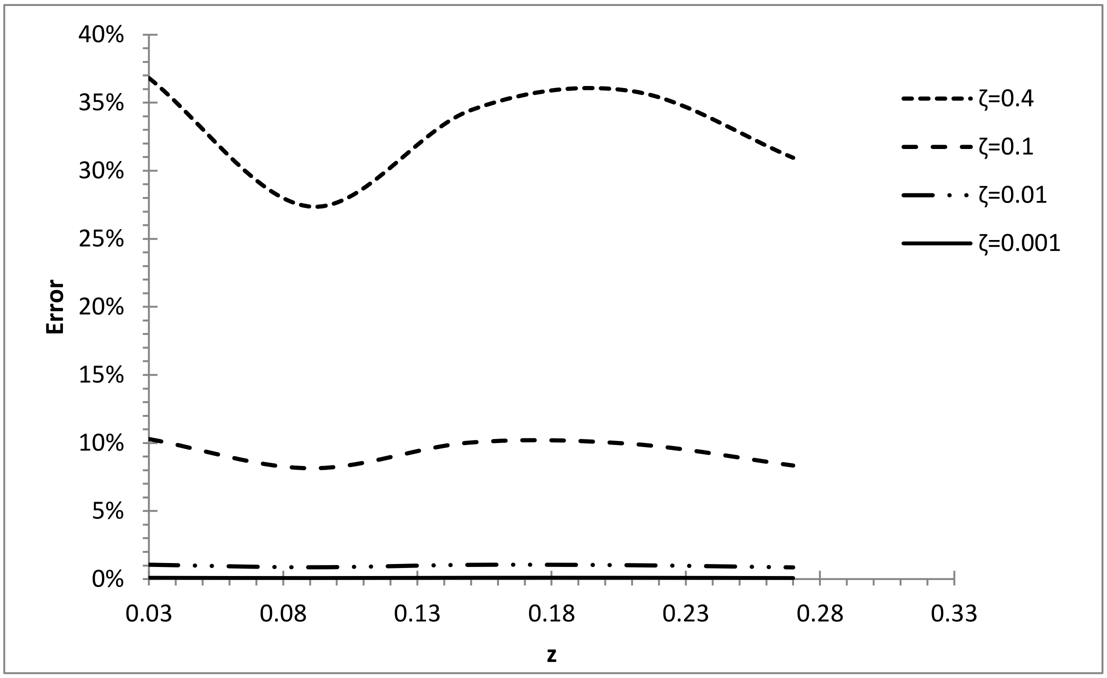

Fig.5. Computer simulation error- Porosity distribution configuration showing a minimum

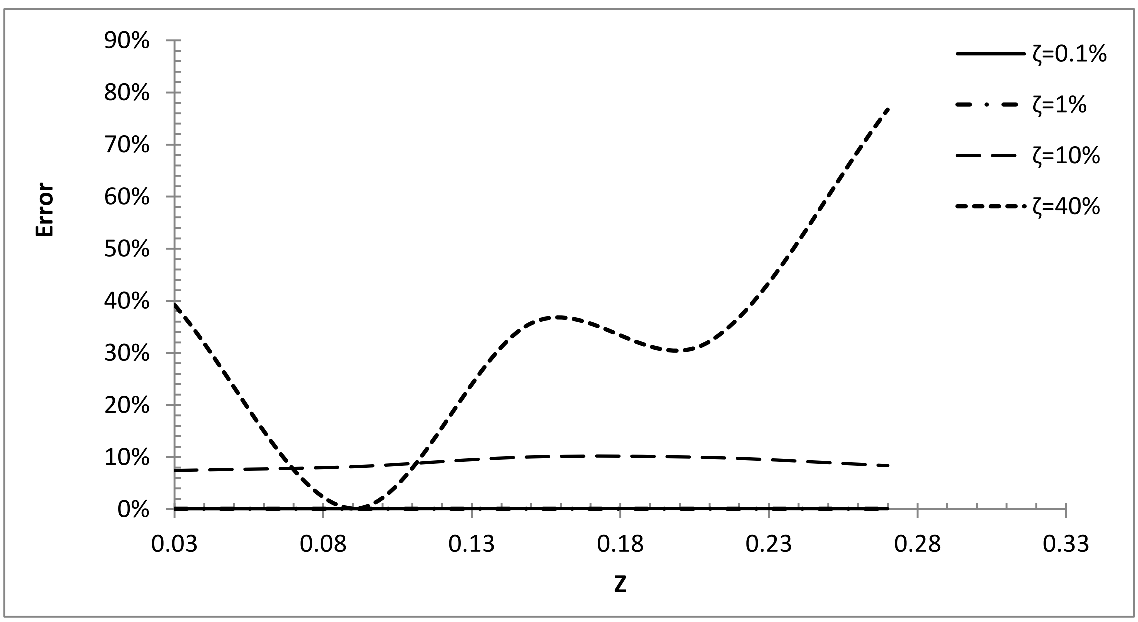

Fig.6. Computer simulation error- Linear porosity distribution

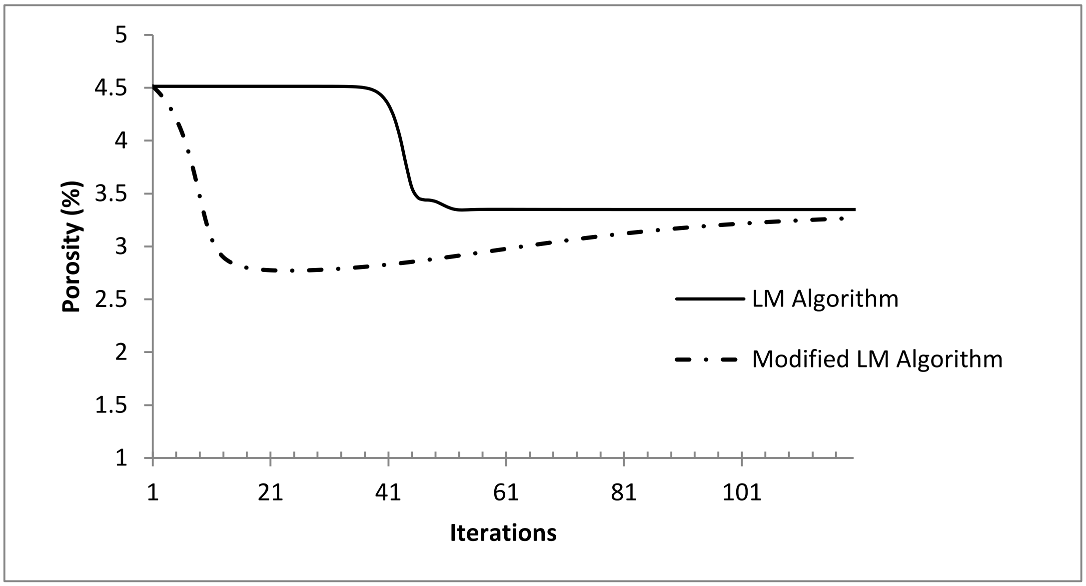

Fig.7. Porosity Estimation

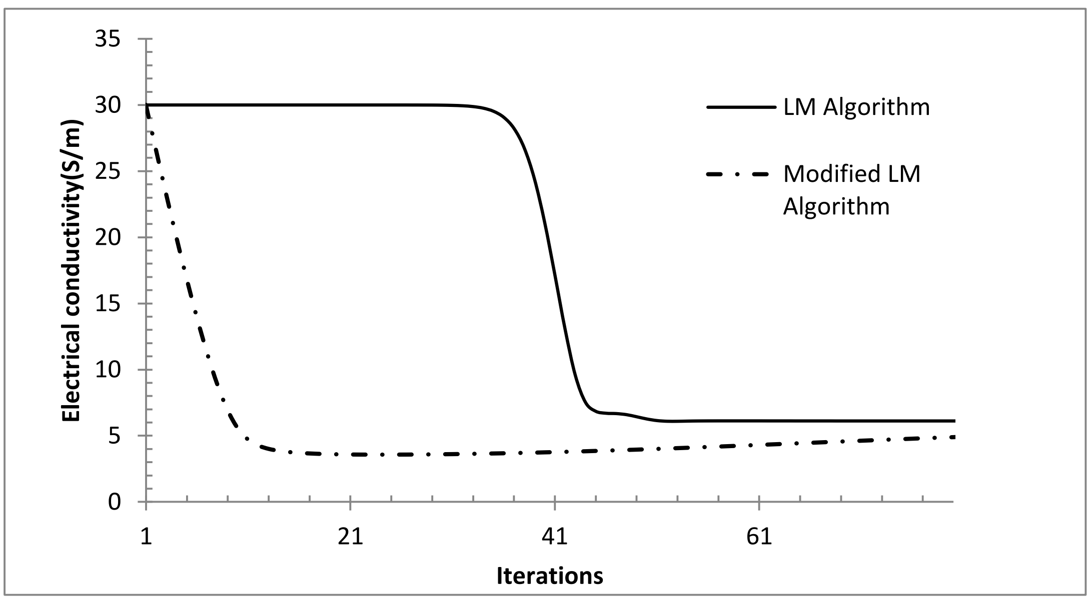

Fig.8. Electrical Conductivity Estimation

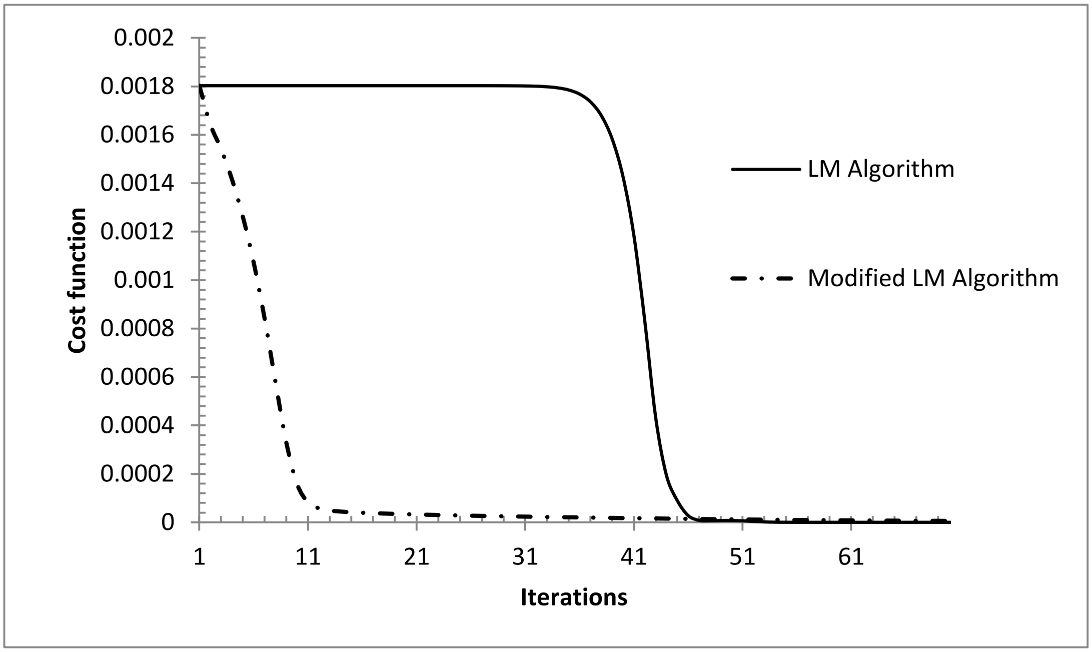

Fig.9. Cost Function

Conclusion:-

In this work, we proposed a theoretical study for the estimation of the porosity distribution in suspension using the ERT technique. The standard technique of Levenberg–Marquardt (LM) is adopted as the estimation procedure to solve successively the inverse ERT problem. The efficiency and stability of this method are analyzed in presence of artificial experimental data. The effects of the measurement errors and the Levenberg Marquardt coefficient on the stability of the solution were analyzed.

The standard LM technique was compared to a modified LM algorithm. Accuracy of the inverse solution provided by this technique is evaluated. The modified algorithm ameliorated the porosity reconstruction using the ERT technique.

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