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**RESEARCH ARTICLE**

# APPROXIMATING FIXED POINT OF TWO GENERALIZED ASYMPTOTICALLY NON EXPANSIVE MAPPING THROUGH KUHFITTING ITERATIVE PROCESS IN HYPERBOLIC SPACES.

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## Abstract

In this paper, we consider an iterative procedure for approximating common fixed points of two generalized asymptotically nonexpansive mappings and we prove Δ-convergence theorems for such mappings in hyperbolic spaces.This will extend the results of Lijuan Zhang and Xian Wang[26] those generalized therein to the case of generalized asymptotically non-expansive mappings.



## Introduction:-

The class of asymptotically nonexpansive mapping, introduced by Goebel and Kirk [3] in 1972, is an important generalization of the class of nonexpansive mapping.They proved that if C is a nonempty closed and bounded subset of a uniformly convex Banach space, then eve- ry asymptotically nonexpansive self mapping of C has a fixed point.

There are number of papers dealing with the approximation of fixed points /common fixed points of asymptotically nonexpansive and asymptotically quasi nonexpansive mappings in uniformly convex Banach spaces using modified Mann and Ishikawa iteration processes and have been studied by many authors.( see, e.g.,[10,11,15,16,17,18,21,22)

The concept of Δ convergence in a general metric space was introduced by Lim 9[9]. In 2008, Kirk and Panyanak [7] used the notion of convergence introduced by Lim [9] to prove in the CAT(0) space and analogous of some Banach space results which involve weak convergence. Further, Dhompongsa and Panyanak [2] obtained Δ convergence theorems for the Picard, Mann and Ishikawa iterations in a CAT(0) space.

A nonlinear framework for fixed point theory is a metric space embedded with a convex structure. The class of hyperbolic spaces, nonlinear in nature, is a general abstract theoretic setting with rich geometrical structure formetric fixed point theory. The study of hyperbolic spaces has been largely motivated and dominated by questions about hyperbolic groups, one of the main objects of study in geometric group theory.

In recent years, Yang and Zhao [25] studied the strong and Δconvergence theorems for total asymptotically nonexpansive nonself-mappingsin CAT(0) spaces. Wan[23] proved some Δ convergence theorems in a hyperbolic space, in which a mixed Agarwal-O'Regan-Sahu type iterative scheme for approximating a common fixed point of totally asymptotically nonexpan-
sive mappings was constructed. Li and Liu [12] modified a classical Kuhfittig iteration algorithm in the general set up of hyperbolic space, and prove a Δ convergence theorem for an implicit iterative scheme.

In this paper, we work in the setting of hyperbolic spaces introduced by Kohlenbach [13], which is more restrictive than the hyperbolic space introduced in Goebel and Kirk [5] and more general than the hyperbolic space in Reich and Shafrir[14].

Concretely, $(X, D, W)$ is called a hyperbolic space if $(X, D)$ is a metric space and $W : X \times X \times [0,1] \to X$ is a function satisfying

(1.1) $d(z, W(x, y, \alpha)) \leq \alpha d(z, x) + (1-\alpha)d(z, y)$;
(1.2) $d(W(x, y, \alpha), W(x, y, \beta)) = |\alpha - \beta| d(x, y)$;
(1.3) $W(x, y, \alpha) = W(y, x, 1-\alpha)$
(1.4) $d(W(x, z, \alpha), W(y, w, \alpha)) \leq (1-\alpha)d(x, y) + \alpha\, d(z, w)$

for all $x, y, z, w \in X$ and $\alpha, \beta \in [0,1]$.A non empty subset $C$ of a hyperbolic space$X$ is convex if $W(x, y, \alpha) \in X(\forall x, y \in X)$ and $\alpha \in [0,1]$. The class of hyperbolic spaces contains normed spaces and convex subsets thereof, the Hilbert ball equipped with the hyperbolic metric [4], Hadamardmanifolds as well as CAT(0) spaces in the sense of Gromov [1].

An important example of a hyperbolic space is the open unit ball $B_H$ in a real Hilbert space H is as follows.
Let $B_H$ be the opeh unit ball in $H$. Then
$K_{B_H}(x, y) = argtanh(1 - \sigma(x, y))^{\frac{1}{2}}$,

where
$\sigma(x, y) = \frac{(1-\|x\|^2)(1-\|y\|^2)}{|1-\langle x, y\rangle|^2}$
for all $x, y \in B_H$ , defines a metric on $B_H$ (also known as Kobayashi distance).

A hyperbolic space X is uniformly convex if for $u, x, y \in X, r > 0$ and$\varepsilon \in (0,1]$, their exists $\delta \in (0,1]$, such that
$d\left(W\left(x, y, \frac{1}{2}\right), u\right) \leq (1-\delta)r$,
provided that $d(x, u) \leq r, d(y, u) \leq r$ and $d(x, y) \geq \varepsilon r$.

A map $\eta: (0, +\infty) \times (0,2] \to (0,1]$ is called modulus of uniform convexity if $\delta = \eta(r, \infty)$ for given $r > 0$ . Besides , $\eta$ is a monotone if it decreases with r, that is,
$\eta(r_2, \varepsilon) \leq \eta(r_1, \varepsilon), cr_2 \geq r_1$ .

Let C be a nonempty subset of a metric space $(X, d)$. A mapping $T: C \to X$ is said to be nonexpansive if
$d(Tx, Ty) \leq d(x, y), \forall x, y \in C$ .

Recall that C is said to be a retraction of $X$ if there exists a continuous map $P: X \to C$ such that $Px = x, \forall x \in C$.A map $P: X \to C$ is said to be retraction if $P^2 = P$. Consequently, if P is a retraction, then $Py = y$ for all $y$ in the range of $P$.
Let C be a nonempty and closed subset of a metric space ($X$, d), A map $P: X \to C$ is a retraction ,
a mapping $T: C \to X$ is said to be

(1 ) asymptotically nonexpansive nonself- mapping2[10] if there exists a sequence $\{k_n\} \subset [1,\infty)$ with $\lim_{n\to\infty} k_n = 1$, such that

$d(T(PT)^{n-1}x, T(PT)^{n-1}y) \leq k_n d(x, y), \forall x, y \in C, n \geq 1$.
(2) A nonself mapping is said to be uniformly L-Lipschitzian if there exists a constant $L \geq 0$ such that
$d(T(PT)^{n-1}x, T(PT)^{n-1}y) \leq Ld(x, y), \forall x, y \in C, n \geq 1$.

(3) Generalized asymptotically nonexpansive (see [19]) if there exist non negative real sequences $\{k_n\}$ and $\{c_n\}$ with $k_n \geq 1$, $\lim_{n\to\infty} k_n = 1$and $\lim_{n\to\infty} c_n = 0$ such that

$d(T(PT)^{n-1}x, T(PT)^{n-1}y) \leq k_n d(x,y)+c_n$ ,for all $x, y \in C$ and $\in N$.
(4) Generalized asymptotically quasi nonexpansive (see [19]) if there exist non negative real sequences $\{k_n\}$ and $\{c_n\}$ with $k_n \geq 1$, $\lim_{n\to\infty} k_n = 1$and $\lim_{n\to\infty} c_n = 0$ such that
$d(T(PT)^{n-1}x, p) \leq k_n d(x,p)+c_n$ ,for all $x \in C$ , $n \in N$ and $\in F(T)$ .

From the definitions above, we know that each nonexpansive mapping is an asymptotically nonexpansive nonself-mapping, and each asymptotically non-expansive nonself-mapping is uniformly L $= sup_{n\geq 1}\{k_n\}$- Lipschitzian.

To prove the results we make use of following basic concepts.Let$\{x_n\}$ be a bounded sequence in hyperbolic space $X$. For $\in X$ , define a continuous functional r(.,$\{x_n\}$) : $X \to [0,+\infty)$ by

r($x$,$\{x_n\}$) =$limsup_{n\to\infty} d(x, x_n)$ .

The asymptotic radius r($\{x_n\}$) of $\{x_n\}$) is given by

r($\{x_n\}$) =inf{ r($x$,$\{x_n\}$) : $x \in X$ , }.
The asymptotic radius $r_C(\{x_n\})$ of $\{x_n\}$ with respect to $C \subset X$ is given by
$r_C(\{x_n\})$ = inf{ r($x$,$\{x_n\}$) : $x \in C$ , }.
The asymptotic center A ($\{x_n\}$ of $\{x_n\}$ is a set,
A($\{x_n\}$ ={ $x \in X$ : r($x$,$\{x_n\}$) = r($\{x_n\}$)}.
The asymptotic center $A_C(\{x_n\})$ of $\{x_n\}$ with respect to $C \subset X$ is given by

$A_C(\{x_n\})$ ={ $x \in C$ : r($x$,$\{x_n\}$) $= r_C(\{x_n\})$ }.
A sequence $\{x_n\}$ in hyperbolic space X is said to Δ-convergence to $x \in X$ , If
$x$ is the unique asymptotic center of $\{u_n\}$for every subsequence $\{u_n\}$ of $\{x_n\}$
In this case, we call $x$ the Δ limit of $\{x_n\}$ .
The purpose of paper is to study an explicit improved Kuhfitting iterative scheme for common fixed points of two asymptotically nonexpansive nonself mappings in hyperbolic spaces. Und- er a limit condition, we obtained a Δ-convergence theorem. This is a development to the resu- lts of [12].

## Preliminary and Lemmas:-

**Lemma 2.1** [6] Let $(X, d, W)$ be a complete uniformly convex hyperbolic space with monot- one modulus of uniform convexity $\eta$ and let $C$ be a nonempty,closed and convex subset of $X$.Then every bounded sequence$\{x_n\}$ in X has a unique asymptotic center with respect to $C$.

**Lemma 2.2** [6,8] Let $(X, d, W)$ be a complete uniformly convex hyperbolic space with mon- one modulus of uniform convexity $\eta$. Let $x \in X$ and $\{\beta_n\}$ be a sequence in [a,b] for some a,b∈ (0,1) . Let $\{x_n\}$ ,$\{y_n\}$ are sequences in $X$ such that $\lim_{n\to\infty} sup d(x_n, x) \leq C$ and $\lim_{n\to\infty} d(W(x_n, y_n, \beta_n), x) = c$ for some $c \geq 0$, then $\lim_{n\to\infty} d(x_n, y_n)$ =0.

**Lemma 2.3** [24] Let $C$ be a nonempty closed convex subset of a uniformly convex hyperbolic space,and let $\{x_n\}$ be a bounded sequence in $C$ such that $A(\{x_n\}) = \{p\}$ and $r(\{x_n\}) = \rho$. If $\{y_m\}$ is another sequence in $C$ such that $\lim_{m\to\infty} sup r(y_m, \{x_n\}) = \rho$ ,then $\lim_{m\to\infty} y_m = p$ .

**Lemma 2.4** [20] Let $\{a_n\}$ and $\{t_n\}$ be two sequences of non negative real numbers satisfying the inequality $a_{n+1} \leq a_n + t_n$ for all n ≥ 1 . If $\sum_{n=1}^{\infty} t_n < +\infty$, then $\lim_{n\to\infty} a_n$ exists.

### Main Results:-

**Theorem 3.1** Let $C$ be a nonempty closed convex subset of a complete uniformly convex hyperbolic space $X$ with monotone modulus of uniform convexity $\eta$, and $P: X \to C$ be the nonexpansive retraction. Let $S_1, S_2:: C \to X$ be two generalized asymptotically nonexpansive nonself mappings with sequence $\{k_n\}, \{l_n\} \subset [1, \infty)$ such that $\sum_{n=1}^{\infty} (k_n - 1) < \infty$ , $\sum_{n=1}^{\infty} (l_n - 1) < \infty$ and $F = F(S_1) \cap F(S_2) \neq \emptyset$.Suppose that $\{\alpha_n\}, \{\beta_n\}$ are real sequences in [a,b] for some a,b ∈ (0,1) . Let $\{x_n\}$ be a sequence generated by the following manner:

$$x_1 \in C \ ,$$
$$y_n = PW(x_n, S_2(PS_2)^{n-1}x_n, \beta_n) \quad (3.1)$$
$$x_{n+1} = PW(y_n, S_1(PS_1)^{n-1}y_n, \alpha_n).$$

Then the sequence $\{x_n\}$ $\Delta$ -converges to a point q ∈ F.

**Proof** We prove the theorem in three steps.

Step 1 .We prove that $\forall p \in F$ , $\lim_{n\to\infty} d(x_n, p)$ and $\lim_{n\to\infty} d(x_n, F)$ exist.Setting $k_n$=1+$u_n$ , $l_n$=1+$v_n$, so $\sum_{n=1}^{\infty} u_n < \infty$ , $\sum_{n=1}^{\infty} v_n < \infty$. Using (1.1)and (3.1), we have that

$$\begin{aligned} d(y_n, p) &= d(PW(x_n, S_2(PS_2)^{n-1}x_n, \beta_n), p) \\ &\le d(W(x_n, S_2(PS_2)^{n-1}x_n, \beta_n), p) \\ &\le (1-\beta_n)d(x_n, p) + \beta_n\, d(S_2(PS_2)^{n-1}x_n, p) \qquad (3.2) \\ &\le (1-\beta_n)d(x_n, p) + \beta_n l_n d(x_n, p) + \beta_n c_n \\ &\le l_n d(x_n, p) + \beta_n c_n \end{aligned}$$

and so

$$\begin{aligned} d(x_{n+1}, p) &= d(PW(y_n, S_1(PS_1)^{n-1}y_n, \alpha_n), p) \\ &\le d(W(y_n, S_1(PS_1)^{n-1}y_n, \alpha_n), p) \\ &\le (1-\alpha_n)d(y_n, p) + \alpha_n\, d(S_1(PS_1)^{n-1}y_n, p) \\ &\le (1-\alpha_n)d(y_n, p) + \alpha_n k_n d(y_n, p) + \alpha_n d_n \\ &\le k_n d(y_n, p) + \alpha_n d_n \qquad (3.3) \\ &\le k_n l_n d(x_n, p) + k_n \beta_n c_n + \alpha_n d_n \\ &= (1+v_n)(1+u_n)\, d(x_n, p) + (k_n \beta_n c_n + \alpha_n d_n) \end{aligned}$$

Since $\sum_{n=1}^{\infty}(u_n + v_n + u_n v_n) < \infty$,we have $\{d(x_{n+1}, p)\}$ is bounded,and then $\{x_n\}$ is also bounded. It implies that there exists a constant $M > 0$ such that $d(x_{n+1}, p) \le M$ for all n≥ $1$. So

$$d(x_{n+1}, p) \le d(x_n, p) + (u_n + v_n + u_n v_n)\, M.$$

Consequently, it follows from Lemma 2.4 that $\lim_{n\to\infty} d(x_n, p)$ and $\lim_{n\to\infty} d(x_n, F)$ exist.

Step-2 We prove that $\lim_{n\to\infty} d(x_n, S_1 x_n) = \lim_{n\to\infty} d(x_n, S_2 x_n)$ =0.

Assume that $\lim_{n\to\infty} d(x_n, p)$ =c≥ $0$. Using (3.2),we have $d(y_n, p) \le l_n\, d(x_n, p) + l_n c_n$.Taking the limsup on both sides in this inequality, we have

$$\limsup_{n\to\infty} d(y_n, p) \le c. \qquad (3.4)$$

In addition, $d(S_1(PS_1)^{n-1}y_n, p) \le k_n d(y_n, p) + d_n$. Taking the limsup on both sides in this inequality, we have

$$\limsup_{n\to\infty} d(S_1(PS_1)^{n-1}y_n, p) \le c \qquad (3.5)$$

From (3.3) we have

$$\begin{aligned} d(x_{n+1}, p) &\le d(W(y_n, S_1(PS_1)^{n-1}y_n, \alpha_n), p) \\ &\le k_n l_n d(x_n, p) + k_n \beta_n c_n + \alpha_n d_n \end{aligned}$$

Since $k_n, l_n \to 1$, $c_n,\ d_n \to 0$ ,n→ ∞ and $\lim_{n\to\infty} d(x_n, p) = c$ ,we have

$$\lim_{n\to\infty} d(W(y_n, S_1(PS_1)^{n-1}y_n, \alpha_n), p) = c. \qquad (3.6)$$

It follows from (3.4)-(3.6) and Lemma 2.2 that

$$\lim_{n\to\infty} \sup \quad d(y_n, S_1(PS_1)^{n-1}y_n) = 0 \qquad (3.7)$$

In addition , $d(S_2(PS_2)^{n-1}x_n, p) \le l_n d(x_n, p) + c_n$ and taking the limsup on both sides in this inequality, we have

$$\limsup_{n\to\infty} d(S_2(PS_2)^{n-1}x_n, p) \le c \qquad (3.8)$$

From (3.3) we have

$$\begin{aligned} d(x_{n+1}, p) &\le (1-\alpha_n)d(y_n, p) + \alpha_n\, d(S_1(PS_1)^{n-1}y_n, p) \\ &\le (1-\alpha_n)d(y_n, p) + \alpha_n\, d(S_1(PS_1)^{n-1}y_n, y_n) + \alpha_n\, d(y_n, p) \qquad (3.9) \\ &\le d(y_n, p) + d(S_1(PS_1)^{n-1}y_n, y_n). \end{aligned}$$

taking the liminf on both sides in this inequality (3.9),by $\lim_{n\to\infty} d(x_n, p)$ =c and (3.7), we have

$$\lim_{n\to\infty} \inf \quad d(y_n, p) \ge c \qquad (3.10)$$

It follows from (3.4) and (3.10) that

$\lim_{n\to\infty} d(y_n,p) = c$ . Using (3.2),this implies that

$$d(y_n,p) \leq d(W(x_n,S_2(PS_2)^{n-1}x_n,\beta_n),p) \leq l_n d(x_n,p)+c_n \quad (3.11)$$

and so

$$\lim_{n\to\infty} d(W(x_n,S_2(PS_2)^{n-1}x_n,\beta_n),p)=0$$

From lemma 2.2 ,we obtain

$$\lim_{n\to\infty} d(x_n,S_2(PS_2)^{n-1}x_n) = 0 \quad (3.12)$$

From $y_n=PW(x_n,S_2(PS_2)^{n-1}x_n,\beta_n)$ and (3.12) we have

$$\begin{aligned} d(x_n,y_n) &= d(x_n, PW(x_n,S_2(PS_2)^{n-1}x_n,\beta_n)) \\ &\leq d(x_n, W(x_n,S_2(PS_2)^{n-1}x_n,\beta_n)) \\ &\leq (1-\beta_n)d(x_n,x_n) + \beta_n d(x_n,S_2(PS_2)^{n-1}x_n) \quad (3.13) \\ &\leq d(x_n,S_2(PS_2)^{n-1}x_n) \to 0, (n\to\infty). \end{aligned}$$

In addition,

$$\begin{aligned} d(x_n,S_1(PS_1)^{n-1}x_n) &\leq d(x_n,y_n) + d(y_n,S_1(PS_1)^{n-1}y_n) \\ &+ d(S_1(PS_1)^{n-1}y_n,S_1(PS_1)^{n-1}x_n) \\ &\leq d(x_n,y_n) + d(y_n,S_1(PS_1)^{n-1}y_n) + k_n d(y_n,x_n) \end{aligned} \quad (3.14)$$

Thus, it follows from (3.7) and (3.13) that

$$\lim_{n\to\infty} d(x_n,S_1(PS_1)^{n-1}x_n) = 0 \quad (3.15)$$

Using (3.1), we obtain that

$$\begin{aligned} d(x_n,x_{n+1}) &= d(x_n,PW(y_n,S_1(PS_1)^{n-1}y_n,\alpha_n)) \\ &\leq d(x_n,W(y_n,S_1(PS_1)^{n-1}y_n,\alpha_n)) \\ &\leq (1-\alpha_n)d(x_n,y_n)+\alpha_n d\Big(x_n,(S_1(PS_1)^{n-1}y_n,\alpha_n)\Big) \\ &\leq (1-\alpha_n)d(x_n,y_n)+\alpha_n d(x_n,y_n)+\alpha_n d(y_n,S_1(PS_1)^{n-1}y_n) \\ &\leq d(x_n,y_n)+\alpha_n d(y_n,S_1(PS_1)^{n-1}y_n) \quad (3.16) \end{aligned}$$

Thus, it follows from (3.7) and (3.13) that

$$\lim_{n\to\infty} d(x_n,x_{n+1})=0. \quad (3.17)$$

Let $L=\sup\{k_n : n \geq 1\}$,$S_1$ is uniformly $L$-lipschitizian.Denote as $(PS_1)^{1-1}$ the identity maps from $C$ onto itself. Thus by the inequality (3.15) and (3.17),we have

$$\begin{aligned} d(x_n,S_1x_n) &\leq d(x_n,x_{n+1}) + d(x_{n+1},S_1(PS_1)^n x_{n+1}) \\ &+ d(S_1(PS_1)^n x_{n+1},S_1(PS_1)^n x_n) + d(S_1(PS_1)^n x_n,S_1x_n) \\ &\leq (1+L)\, d(x_n,x_{n+1}) + d(x_{n+1},S_1(PS_1)^n x_{n+1}) \\ &+ d(S_1(PS_1)^{1-1}(PS_1)^n x_n,S_1(PS_1)^{1-1}x_n) \\ &\leq (1+L)\, d(x_n,x_{n+1}) + d(x_{n+1},S_1(PS_1)^n x_{n+1}) + L\, d((PS_1)^n x_n,S_1x_n) \\ &\leq (1+L)\, d(x_n,x_{n+1}) + d(x_{n+1},S_1(PS_1)^n x_{n+1}) \\ &+ L\, d(S_1(PS_1)^{n-1}x_n,x_n) \to 0(n\to\infty). \end{aligned}$$

Similarily, we may show that $\lim_{n\to\infty} d(x_n,S_2x_n)=0 \quad (3.18)$

Step-3.We prove that $\{x_n\}$ Δ-converges to a point $q \in$F. Since $\{x_n\}$ is bounded, by Lemma 2.1, it has a unique asymptotic center $A_C(\{x_n\}) =\{q\}$.If $\{w_n\}$ is any sequence of $\{x_n\}$ with $A_C(\{w_n\})=\{w\}$), then by (3.18) we have

$$\lim_{n\to\infty} d(w_n,S_2w_n)=0 \quad (3.19)$$

We claim that $w\in F$.In fact, for all m,n$\geq 1$,

$$\begin{aligned} d(S_1(PS_1)^{m-1}w,w_n) &\leq d(S_1(PS_1)^{m-1}w,S_1(PS_1)^{m-1}w_n) \\ &+ d(S_1(PS_1)^{m-1}w_n,S_1(PS_1)^{m-2}w_n) + \ldots\ldots + d(S_1w_n,w_n) \\ &\leq k_m d(w,w_n) + d_m + Ld\ (S_1w_n,w_n) + \ldots\ldots + d(S_1w_n,w_n) \\ &\to 0(n\to\infty). \end{aligned}$$

Taking lim sup on both sides of the above estimate and using and using (3.19),we obtain that

$$r\left(S_1(PS_1)^{m-1}w,\{w_n\}\right)=\limsup_{n\to\infty} d(S_1(PS_1)^{m-1}w, w_n)$$
$$\le \limsup_{n\to\infty} d(w,w_n)= r\left(w,\{w_n\}\right).$$

This implies that

$$\lim_{m\to\infty} r\left(S_1(PS_1)^{m-1}w,\{w_n\}\right)= d(w,w_n).$$

By Lemma 2.3, we have

$$\lim_{m\to\infty} S_1(PS_1)^{m-1}w = w \quad .$$

Because $S_1$ is uniformly continuous , we have

$$S_1 w = S_1 \lim_{m\to\infty} S_1(PS_1)^{m-1}w = S_1 \lim_{m\to\infty} PS_1(PS_1)^{m-1}w = \lim_{m\to\infty} S_1(PS_1)^m w = w.$$

Consequently, $w \in F(S_1)$ , Using the same method, we have prove that $w \in F(S_2)$ and $w \in F$.
By the uniqueness of asymptotic center,we have $w = q$. It implies that $q$ is the unique asymptotic center of $\{w_n\}$ for each subsequence $\{w_n\}$ of $\{x_n\}$, that is $\{x_n\}$ Δ-converges to a point $q \in$F.

Remark 3.1

(a) Theorem 3.1 removes the assumption about $0 < b(1-a) < \frac{1}{2}$ in [4]

(b)Theorem 3.1 generalizes the results of [4] from a two asymptotically nonexpansive nonself mappings to generalized asymptotic mappings.

## References:-


1. M. Bridson, A. Haeiger, Metric Spaces of Non-Positive Curvature.Springer, Berlin, 1999.
2. S. Dhompongsa and B. Panyanak, On Δconvergence theorem in CAT(0) spaces, Comput. Math. Appl. 56(10) (2008), 2572–2579.
3. K. Goebel and W.A. Kirk, A fixed point theorem for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc. 35 (1972), 171-174.
4. K. Goebel, S. Reich, Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings. Dekker, New York, 1984
5. K. Goebel, W. A. Kirk, Iteration processes for nonexpansive mappings. In:Singh, SP, Thomeier, S, Watson, B (eds.) Topological Methods in Nonlinear Functional Analysis Contemp Math, vol. 21, pp. 115-123. Am. Math.
6. R. Khan, Fukharuddin, H, Khan, MAA, An implicit algorithm for two finite families of nonexpansive maps in hyperbolic space, Fixed Point Theory Appl, 2012, Article ID 54, 2012.
7. W. A. Kirk and B. Panyanak, A concept of convergence in geodesic spaces, Nonlinear Anal. 68(12) (2008), 3689–3696.
8. Leustean, L: Nonexpansive iterations uniformly covex W-hyperbolic spaces.In:Leizarowitz, A, Mordukhovich, BS, Shafrir, I, Zaslavski, A(eds.) Nonlinear Analysis and Optimization I.
9. T. C. Lim, Remarks on some fixed point theorems, Proc. Amer. Math. Soc. 60 (1976),179–182.
10. Q.H. Liu, Iterative sequences for asymptotically quasi-nonexpansive mappings, J. Math. Anal. Appl. 259 (2001), 1-7.
11. Q.H. Liu, Iterative sequences for asymptotically quasi-nonexpansive mappings with error member, J. Math. Anal. Appl. 259 (2001), 18-24.
12. Y. Li, H. B. Liu,Δ -Convergence analysis of improved Kuhfittig iterative for asymptotically nonexpansive nonself-mappings in W-hyperbolic spaces, J Inequal Appl, 2014, 2014: 303
13. U. Kohlenbach, Some logical metatheorems with applications in functional analysis, Trans Am Math Soc, 357(2005) 89-128.
14. S. Reich, I. Shafrir, Nonexpansive iterations in hyperbolic spaces.Nonlinear Anal, 15(1990), 537-558.
15. B.E. Rhoades, Fixed point iteration for certain nonlinear mappings, J. Math. Anal. Appl. 183(1994), 118-120.
16. G.S. Saluja, Strong convergence theorem for two asymptotically quasinon expansive mappings with errors in Banach space, Tamkang J. Math. 38(1) (2007), 85-92.
17. G.S. Saluja, Convergence result of $(L,\alpha)$uniformly Lipschitz asymptotically quasi-nonexpansive mappings in uniformly convex Banach spaces, jnanabha 38 (2008), 41-48.
18. N. Shahzad, A. Udomene, Approximating common fixed points of two asymptotically quasi-nonexpansive mappings in Banach spaces, Fixed Point Theory and Applications, 2006 (2006), Article ID 18909
19. N. Shahzad, H. Zegeye,. – Strong convergence of an implicit iteration process for a finite family of generalized asymptotically quasi-nonexpansive maps, Appl. Math. Comput., 189 (2007), 1058–1065

20. K. K. Tan, H. K. Xu, Approximating fixed points of nonexpansive mapping by the Ishikawa iteration process, J Math Anal Appl, 178 (1993), 301-308.

21. K.K. Tan and H.K. Xu, Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process, J. Math. Anal. Appl. 178, 301-308, 1993.
22. K.K. Tan and H.K. Xu, Fixed point iteration processes for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc. 122(1994), 733-739.
23. L.Wan, 4-Convergence for mixed-type total asymptotically non expansive mappings in hyperbolic spaces, J Inequal Appl, 2013, Article ID 553, 2013.
24. L. Wang, S. S. Chang, Z. Ma, Convergence theorem for total asymptotically nonexpansive nonself mappings in CAT(0) space, J Inequal Appl,2013, Article ID 135, 2013.
25. L. Yang, F. H. Zhao, Strong and 4-convergence theorems for total asymptotically nonexpansive nonself mappings in CAT(0) spaces, J Inequal Appl, 2013, Article ID 557, 2013.
26. Lijuan Zhang1 and Xian Wang, Δ Convergence for Common Fixed Points of Two Asymptotically Nonexpansive Nonself Mappings in Hyperbolic Spaces International Journal of Mathematical Analysis Vol. 9, 2015, no. 8, 385 – 393