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RESEARCH ARTICLE

THE BENEFITS OF PROBLEM POSING IN THE LEARNING OF MATHEMATICS: A SYSTEMATIC REVIEW.

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Abstract

Problem posing has received considerable attention recently by researchers in mathematics education. Researchers in mathematics education have reported that problem posing is an effective strategy for teaching and learning mathematics. This study was carried out to bring to limelight the benefits of problem posing intervention in learning mathematics. The study was a systematic review of empirical studies on the effects of problem posing instructional strategy in the learning of mathematics. Search for the studies was both electronically and manually, and many studies were found, but only 16 studies met the requirements for inclusion, and the participants involved in these studies were 1871. The studies that met the requirements were examined in depth, especially in terms of the title, study design, participants and findings of each study. It was revealed that all the studies reported significant improvement in the learning outcomes studied. The learning outcomes related to all the 3 learning domains of Bloom's taxonomy (cognitive, psychomotor, and affective). Problem posing instructional strategy was reported to have impacted positively on the students' learning of mathematics. Problem posing instruction was shown to support knowledge-based, skill-based, ability-based and attitude/belief-based learning. It was advocated that problem posing be made an integral part of the mathematics curriculum.

Introduction:-

Researchers in mathematics education view problem posing as a significant mental activity in mathematics (e.g. Rosli, Capraro & Capraro, 2014; Sengul & Katranci, 2012; Lavy & Shriki, 2007; Pittali, Christou, Mousoulides & Pitta-Pantazi, 2004). In fact, some researchers referred to it as being at the heart of mathematics (Kojima,Miwa & Matsui, 2015). Problem posing is a research topic in mathematics that is currently receiving the attention of many mathematics educators. Researchers such as Akay and Boz (2008) and Lavy and Shriki (2007) pointed to problem posing as an important strategy for teaching and learning mathematics, while others consider it an important skill that helps in solving problems (Kojima, Miwa & Matsui, 2015; Guvercin & Verbovskiy, 2014; Silver, 1994). Akay and Boz (2008) pointed a close relation between problem posing as an approach and academic success.

There is a general consensus of researchers in Mathematics education that problem posing is the creation of new problems and the recreation of given problems. For instance, Stoyanova and Ellerton (1996) see posing problem as

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the process by which students create personal explanations of concrete situations and formulate such situations into sensible mathematical problems. Similarly, Pittalis, Christou, Mousoulides and Pitta-Pantazi (2004) define posing problems as creation of new problems and reformulation of given problems. This study considers problem posing as the creation of new problems and changing certain parameters of existing problems.

Posing problems in the learning of mathematics:-

The benefits of posing problems have been pointed out by many researchers in mathematics education. For instance, Pittalis et al (2004) pointed out that students could solve mathematical word problems as a result of using problem posing as an instructional strategy. Similarly, English (1997) found that problem posing afforded teachers the opportunity to comprehend students' thinking about concepts and processes in mathematics. Research has also indicated that problem posing improves students' thinking and problem solving skills (Guvercin & Verbovskiy,2014), attitudes and confidence in mathematics (Guvercin, Cilavdaroglu &Savas,2014). Similarly, according to Lavy and Shriki (2007) and Stoyanova (1999) posing problem fosters flexible thinking, and enhances students' problem solving skills, reasoning and reflection. Problem posing is a necessary part of learning mathematics (Sengul & Katranci, 2012; Lavy & Shriki, 2007). Kojima, Miwa and Matsui (2015) say it is an important activity in mathematics education, and lies at the heart of mathematical activity. They assert that problem posing is a necessary skill for solving problems. Singer, Ellerton and Cai (2013) assert that posing problem raises creativity in students and adds to mathematics talent and independent learning. Posing problem is viewed as an investigative-oriented instruction, and also as a distinct feature of creative activity (Silver, 1994). In fact, Ticha and Hospesova (2009) point out the potential benefit of problem posing as the diagnosis of teacher-students' subject teaching knowledge. In other words, problem posing can detect the nature of the student's mathematics learning problem.

Similarly, Ticha and Hospesova (2006) assert that problem posing is one of the potential options for pre-service primary mathematics teachers to acquire professional aptitude in mathematics. According to Ticha and Hospesova (2009), experience has revealed that pre-service primary mathematics teachers' subject teaching competence is dependent on their ingenuity to create mathematical problems. This, therefore, underscores the importance of engaging in activities that are targeted at problem posing as an instructional strategy in the teaching and learning of mathematics. This importance has been pointed to by many mathematics education researchers such as English (1997), Pittalis et al (2004), Akay and Boz (2008), and Cankoy and Darbaz (2010). These mathematics educators and a host of others have stressed the desire and importance for students to acquire the ability to pose problems. Ticha and Hospesova (2009) opine that problem posing should not be seen only as a goal of teaching, but also as a means of teaching. Problem posing relates to competence in mathematics and, this is the reason for emphasizing that students be given the opportunity to create their own problems in the learning of mathematics. There is a consensus among mathematics educators that problem posing enriches student's learning of mathematics, and guides the students to a deeper understanding of mathematical concepts (e.g. Abu-Elwan, 2002; Guvercin & Verbovskiy, 2014; Haghverdi & Gholami, 2015).

This study focused on searching the literature to highlight the benefits of problem posing in the learning of mathematics. It is a systematic review of experimental studies that have been conducted on the effect of problem posing as an instructional strategy in the teaching and learning of mathematics. Many experimental studies have been carried out in mathematics with problem posing as an instructional strategy (e.g. Abu-Elwan, 2002; Akay & Boz, 2008; Priest, 2009; Kesa, Kaya & Guvercin, 2010;Guvercin & Verbovskiy, 2014; Kojima, Miwa & Matsui, 2015). And a variety of outcomes have resulted from such studies. There is therefore the need for a systematic review of the literature to highlight these results, especially that the studies are conducted at various locations on the continent.

Purpose of the study:-

Research has shown that there are many studies in mathematics education that used problem posing as an instructional strategy to determine students' achievement in mathematics, and so the desire to investigate the benefits of problem posing is timely. Research evidence has indicated improvements on students' learning of mathematics through problem posing intervention. The purpose of this study was to answer the question: do students benefit more when problem posing is used as an instructional strategy in the teaching and learning of mathematics? The study employed a systematic review of empirical studies to investigate the effect of problem posing on students' learning of mathematics. This was done by considering experimental studies only.

Method:-

Criteria for the selection of studies to be included:-

The first step was to determine the keywords to be used for searching on the website. These words or phrases included, *problem posing in mathematics*, *effect of problem posing in mathematics*, *problem solving and problem posing in mathematics*, *experimental studies in mathematical problem posing*, *investigation of problem posing in mathematics* and *effectiveness of problem posing*. The next step was a manual search in the Journals of Mathematics Education and other Multidisciplinary Journals. The search was restricted to only journals written in English Language. Only experimental studies that indicated clearly the research design, respondents/population, and the results were considered as meeting the requirements for inclusion. However, the literature review in the background was not restricted to review of experimental studies only.

Upon these searches, many studies relating to problem posing in mathematics were found. In fact 3,815 studies resulted from these multiple searches. But most of the studies found did not meet the requirements for inclusion. Majority not experimental, some were duplicates, others inaccessible or not traceable, and a few others not published in English Language. Therefore, all those studies that did not meet the requirements for inclusion were discarded. The experimental studies were examined in depth with respect to the eligibility criteria for inclusion. Applying these criteria only 16 studies were retained, and the analysis is shown in Table 1.

Table 1:- Experimental Studies on Effects of Problem Posing.

Author and date	Title of study	Participants	Results/findings
Abu-Elwan (2002)	Effectiveness of problem posing strategies on prospective mathematics teachers' problem performance	Student-teachers 50	1.Problem posing significantly improved the mathematics achievement of the experimental group 2. Experimental group exhibited significantly higher level of problem posing performance compared to the control group.
Akay & Boz (2008)	The effect of problem posing oriented Calculus-II instruction on academic success	Pre-service teachers 79	1.Problem posing approach increased the academic success of the experimental group 2. A close and positive relation between academic success and problem posing approach was found.
Akay & Boz (2010)	The effect of problem posing oriented analysis II course on the attitudes toward mathematics and mathematics self-efficacy	Elementary pre-service teachers 82	1. Significant differences were found in the participants' attitudes toward mathematics and mathematics self-efficacy between the experimental and control groups. 2. The experimental group showed positive attitude toward mathematics.
Cankoy & Darbaz (2010)	Effect of a problem posing based problem solving instruction of understanding problem (UP)	3rd grade students 53	1. Experimental group students were better than the control group students in terms of UP scores in all dimensions (rephrasing, visualization and qualitative reasoning). 2. Experimental group was better than control group in finding missing information in a given problem and the contradictions of a problem.

Demir (2005)	The effect of instruction with problem posing on tenth grade students' probability achievement and attitudes toward probability	Tenth Grade 82	1. The students taught by problem posing instruction had significantly greater probability achievement than students taught by traditional method. 2. There was a significant difference between the mean scores of the students who received instruction with problem posing and those who were taught without in terms of attitudes toward probability.
English (1997)	The development of fifth grade children's problem posing abilities	Fifth Grade children 27	1.Substantial development was exhibited by children who participated in the program
English (1998)	Children's problem posing writing formal and informal contexts.	Year 3 in elementary school 54	1. Children's participation in the problem posing program led to greater diversity in the problems posed.
Fetterly (2010)	An exploratory study of the use of a problem posing of pre-service elementary education teachers' mathematical creativity, beliefs, and anxiety.	Elementary Pre-service teachers 32	1. It was found that mathematical creativity can be fostered and sustained by problem posing. 2. Problem posing impacted mathematics beliefs positively.
Guvercin & Verbovskiy (2014)	The effect of problem posing tasks used in mathematics to mathematics academic achievement and attitudes toward mathematics	High school students 54	1. Problem posing instruction significantly increased students' mathematical academic achievement. 2. Gave the students more confidence and positive attitudes toward mathematics. 3. Problem posing encouraged students to be creative, divergent and flexible in their thinking.
Guvercin, Cilavdaroglu &Savas (2014)	The effect of problem posing instruction on ninth grade students' mathematics academic achievement and retention	Grade 9 students 60	1. Problem posing instruction increased students' academic achievement. 2. Higher positive attitudes toward mathematics. 3. More confidence and positive attitudes through active involvement
Haghverdi & Gholami (2015)	A study of the effect of using "What if Not" strategy in posing geometry problems	High school students 29	1.Problem posing led to deeper understanding of geometrical concepts 2. Problem posing strengthened the interconnections between relevant geometrical concepts 3. Improved students' skills
Kesan, Kaya & Guvercin (2010)	The effect of problem posing approach to the gifted student's mathematical abilities	Grade eight students 64	1. Increased and fostered mathematical abilities. 2. Enhanced interaction between teacher

			and student, and makes student active in the lesson.
Kojima, Miwa & Matsui (2015)	Experimental study of learning support through examples in mathematical problem posing	Undergraduates 219	1. Problem posing benefited the experimental group more than the control group (i.e. experimental group posed problems with novel solutions). 2. Found that learning by posing problems was more effective.
Priest (2009)	A problem posing intervention in the development of problem solving competence of underachieving middle-year students	Year seven students 31	1.Participation in problem posing activities facilitated the re-engagement of disengaged middle-year mathematics students 2. Participation in problem posing resulted in improved problem solving competence. 3. There was increased integration of prior knowledge with new knowledge.
Walsh (2016)	Pre-service primary teachers' understanding of mathematical problem posing and problem solving: Exploring the impact of a study intervention	Pre-service primary teachers 415	1.Improved students' conception of what constituted a mathematical problem
Xia , Lu & Wang (2008)	Research on mathematics instruction experiment based on problem posing	Junior high school and grade nine students 540	1. Found significant effects on students' interest in learning mathematics. 2. Problem posing improved students' ability to pose problems. 3. Enhanced students' mathematics learning ability.

Results and Discussion:-

Description of the selected studies:-

A total of 16 experimental studies that made the criteria for inclusion in this study were reviewed as shown in Table 1. Nine studies were published between 2010 and 2016. Five studies published between 2000 and 2009, one study published in 1997 and one in 1998. These studies involved 1871 participants, made up of pre-service teachers, high school students, middle and lower grade students. All the studies employed experimental designs. Three studies used matching-only pretest-posttest control group design. Two used posttest only control group design, and the remaining eleven studies used quasi-experimental non-equivalent groups, and all were pretests-posttests control group design. The instruments used were either researcher constructed or adoption/adaption from the literature. The research targets of these studies were either on performance in mathematics, mathematics achievement in academics or attitudes and beliefs toward mathematics. The researchers were interested in determining the impact of problem posing as an instructional strategy on the performance of students, the achievements of students and attitudes of students toward mathematics. The summary of the outcomes or results of their findings is presented in Table 1. It can be seen from Table 1 that all the studies presented showed positive improvements in either performance in mathematics, achievement in mathematics or attitudes toward mathematics using problem posing as an intervention in the teaching and learning of mathematics.

Eight studies focused on effect of problem posing on mathematics performance and achievement (e.g. Abu-Elwan, 2002; Akay & Boz, 2008; Demir, 2005;Guvercin, Cilavdaroglu &Savas, 2014), and findings from these studies showed significant improvements in students' performance and achievement in mathematics. Two studies showed

that problem posing could foster and sustain mathematical creativity, enhance divergent and flexible thinking in students. In a study titled, 'Problem posing as a means for developing mathematical knowledge of prospective teachers', Lavy and Shriki (2007) pointed out that problem posing enhanced mathematical creativity, and this was corroborated by Singer, Ellerton and Cai (2013). They said that problem posing enhances creativity, adds to mathematical ability and learning independence.

Regarding attitudes and beliefs toward mathematics, six of the studies reviewed indicated that problem posing as an intervention impacted positively on the attitudes and beliefs of students toward mathematics (e.g. Xia, Lu & Wang, 2008; Akay & Boz, 2010; Demir, 2005; Fetterly, 2010; Guvercin & Verbovskiy, 2014). Pittalis, et al (2004) had pointed out that problem posing increased students' attitudes positively toward mathematics. Furthermore, the review revealed that problem posing helped students to understand the concept of what a 'problem' is, and to find missing information in the given problems. A study by Cankoy & Darbaz (2010) revealed that the participants in the treatment condition were better off than those in the control condition in terms of problem understanding. And also, the experimental group was better in identifying missing information in given problems than the control group. These authors' focus was determining the effect of a problem posing based problem solving instruction of understanding problem. The need for a problem solver to understand the problem he/she is to solve is imperative; and in fact, it is the first step toward a successful solution. The students were not only able to find missing information in a given problem, but also could detect contradictions of a problem. Walsh (2016) focus was on understanding mathematical problem posing and problem solving, using problem posing intervention. The study findings revealed that the problem posing intervention improved students' conception of what constituted a mathematical problem.

The study of Haghverdi and Gholami (2015) focused on skills in mathematics. The findings of their studies showed that problem posing improved students' skills in mathematics significantly. In fact, besides the improvement of skills, problem posing instruction strengthened interconnections between relevant geometrical concepts, and led to a deeper understanding of geometrical concepts (Haghverdi & Gholami, 2015). Another benefit of problem posing according to Kesan, Kaya & Guvercin (2010), is that it enhanced interaction between teachers and students, and this led to students being active in the lesson. Furthermore, an increase in the integration of prior knowledge with new knowledge was what Priest (2009) found. He also reported that problem posing activities facilitated the re-engagement of disengaged middle-year mathematics students. His study was on problem posing intervention in the development of problem solving competence of underachieving middle-year students. The intervention was reported to have improved the students' problem solving competence.

Conclusion:-

The findings of the empirical studies examined in this study can be grouped or categorized into the 3 Bloom's taxonomy of learning domains (cognitive, psychomotor and affective domains). The studies revealed that problem posing impacted positively on the following learning outcomes viz- knowledge-based, ability-based, skill-based and attitude/interest and belief-based. This shows clearly that every aspect of learning mathematics can be supported by problem posing instruction. For instance, the studies by Abu-Elwan (2002), Akay and Boz (2008), Cankoy and Darbaz (2010),Guvercin and Verbovskiy (2014), and Walsh (2016) indicated that problem posing intervention supported knowledge-based learning outcome. Similarly, studies by English (1997 & 1998), Abu-Elwan (2002), Xia, Lu, and Wang (2008), Fetterly (2010), and Kojima, Miwa and Matsui (2015) revealed that problem posing instruction impacted positively on ability-based learning outcome. Skill-based learning outcome was seen to have improved significantly through the use of problem posing (Abu-Elwan, 2002; English, 1997; Priest, 2009). Problem posing instructional strategy also improved participants attitudes/interest and beliefs toward mathematics (Demir, 2005; Akay & Boz, 2010; Guvercin, Verbovskiy, 2014 Guvercin, Cilavdaroglu & Savas, 2014).

It could be said with every confidence that, if properly handled, problem posing instructional strategy can be one of the most effective strategies in the teaching and learning of mathematics. This is because not only students benefit from problem posing; but teachers also do. Problem posing affords teachers the opportunity to get an insight into students' thinking processes. Problem posing revealed that mathematics anxiety and fears could be reduced significantly in students. It encourages the use of diverse representations, building of a knowledge network, development of creativity, enhancement of attitude toward mathematics and increment in self-confidence. It can safely be advocated that problem posing be an integral part of the mathematics curriculum, considering the role it can play in the teaching and learning of the subject.

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