

RESEARCH ARTICLE

MULTI OPTIMIZATION OF FRICTION STIR WELDING OF ALUMINIUM AA 6061 ALLOY USING TAGUCHI GREY CO RELATION

Tuniki Omkar¹ and Batte Jai Rathan²

- 1. Department of Mechanical Engineering, Maturi Venkata Subba Rao Engineering College, Hyderabad, India
- 2. Department of Automobile Engineering, Maturi Venkata Subba Rao Engineering College, Hyderabad, India

Manuscript Info

Abstract

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Key words:-

Design of Experiments, Orthogonal Array, Taguchi, Wire Electro Discharge Machining, Stainless Steel 440C, WEDM, ANOVA, Grey Correlation It is often observed that the parameter tables for machining provided by machine tool manufacturers fail to align with operator requirements and, at times, do not offer efficient guidelines as per the expectations of manufacturing engineers. Therefore, selecting suitable machining parameters is critical for achieving the desired output in any machining process. This study focuses on determining the optimal parametric settings during Friction Stir Welding (FSW) of Aluminium AA 6061 alloy. The process parameters considered include welding speed, axial force, and tool rotational speed, while the quality characteristics studied are tensile strength and tensile elongation. To optimize these parameters, the Taguchi method was employed to design the experiments. An $(L9 (3^3))$ orthogonal array was selected based on insights from pilot experiments and a thorough literature review. Significant process parameters influencing machining performance were identified using Analysis of Variance (ANOVA) and F-test values in conjunction with experimental results. Verification of the quality improvements achieved using these optimized parameters was performed, and the findings showed enhancements compared to the original setup parameters. Furthermore, linear regression models were developed to establish relationships between machining performance and the selected parameters. In addition to optimizing each parameter individually, multi-response optimization was conducted using Taguchi-Grey relational analysis to consider the combined effects of the process parameters. The results demonstrate an overall improvement in the quality characteristics of FSW for Aluminium AA 6061, validating the effectiveness of the optimized parametric settings.

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Introduction:-

Friction Stir Welding (FSW) is a solid-state welding process where two components are joined by rubbing them together at a controlled rotational speed to induce friction. The heat generated through friction causes the materials to reach a plastic state, allowing them to bond as they are forced together under lateral pressure, referred to as "upset." This process creates a bond through the intermingling of plasticized material layers from both components, resulting in new, combined material layers [1].

Corresponding Author: - Tuniki Omkar

Address:- Department of Mechanical Engineering, Maturi Venkata Subba Rao Engineering College, Hyderabad, India.

FSW has the potential to replace conventional welding methods and one-piece construction as an economical and efficient welding process. It offers several advantages, including enhanced design flexibility, improved strength, and significant cost savings. Compared to conventional welding techniques, FSW provides superior strength and reliability while enabling innovative design possibilities.

The Aluminium-Magnesium-Silicon alloy AA 6061, a heat-treatable wrought alloy, exhibits excellent welding characteristics among high-strength Aluminium alloys. This alloy is widely used in marine frames, pipelines, storage tanks, and aircraft applications. Unlike traditional fusion welding processes, FSW minimizes common welding defects such as large distortions, solidification cracking, porosity, and oxidation, making it a preferred choice for structural alloy joining [2].

To achieve high-quality welded joints, it is crucial to optimize the process parameters. This study focuses on an experimental investigation of FSW for Aluminium AA 6061 alloy, emphasizing tensile strength and tensile elongation as quality characteristics. The experimentation was conducted using the Taguchi method, considering welding speed, axial force, and tool rotational speed as key process parameters.

Methodology:-

In this study, 6 mm thick plates of 6061 Aluminium alloy were used as the base metal. The chemical composition of the alloy and its properties are detailed in Table 1 and Table 2, respectively. A butt joint configuration was employed for the welding process, which was carried out using a vertical milling machine. The process parameters under investigation—welding speed, axial force, and tool rotational speed—were selected based on a thorough literature survey. The respective parameter levels were determined through preliminary experiments to ensure optimal experimental conditions. Each experiment was conducted three times, and the average values of the quality characteristics were recorded to account for variability in the results.

Table 1:- Chemical composition of base metal (% wt).

Material	Mg	Fe	Cu	Cr	Si	Al
6061-T6	0.78 to	0.32	0.24	0.19	0.3 to	Remaining
	1.19				0.7	

The experimental investigation aims to optimize the FSW process [5] to improve the quality characteristics of tensile strength and tensile elongation. Several studies have demonstrated techniques for selecting optimal parametric values to enhance tensile strength [3] and tensile elongation [4]. The Taguchi method has been extensively used in determining the process parameters due to its effectiveness. This method employs Orthogonal Arrays (OAs) [6] for experiment design, offering simplicity and adaptability as its key advantages. It allows for the extraction of essential information with the minimum number of trials while maintaining precision and reproducibility of the results.

Property	Metric
Density	2.69 g/cc
Brinell Hardness	95
UTS	309 MPa
Modulus of elasticity	69 GPa
Poisson's ratio	0.329
Thermal conductivity	170 W/mK

To evaluate the performance characteristics under optimal machining parameters, a specially designed experimental procedure is essential. A full factorial experimental design considers all possible combinations of factors and levels for a given setup, ensuring comprehensive coverage. However, as the number of factors and levels increases, the total number of experiments grows significantly, making this approach impractical due to financial constraints and time requirements. To address this challenge, Taguchi's orthogonal arrays are employed, which significantly reduce the number of experiments needed while maintaining the reliability of the results [9].

Table 3:- Process parameters and their levels.

Notation	Factor	Unit	Level 1	Level 2	Level 3
А	Welding speed	mm/min	59.8	70.8	99.8
В	Axial force	kN	4.7	5.8	6.9
С	Tool rotational	RPM	899	1000	1099
	speed				

The experiment involves three process parameters, each at three levels. Each process parameter contributes two degrees of freedom, resulting in a total of six degrees of freedom. The interaction effects between the parameters are neglected in this study [6][8].

When selecting an orthogonal array, it is essential to ensure that the degrees of freedom of the array are greater than or equal to those of the process parameters [10]. An L9 array, which has eight degrees of freedom (9 - 1 = 8), satisfies this criterion. Since the process parameters in this study contribute only six degrees of freedom, it is appropriate to use an L9 orthogonal array for the experiments. The experimental layout for the process parameters based on the L9 array is provided in Table 4.

 Table 4:- Experimental layout considered.

Experiment	Α	В	С
1 (Initial setup)	60	5	900
2	60	6	1000
3	60	7	1100
4	80	5	1000
5	80	6	1100
6	80	7	900
7	100	5	1100
8	100	6	900
9	100	7	1000

Experimental Results

The experiments are conducted according to the arrangement of the orthogonal array presented in Table 4. For each run, three sets of experiments are performed, and the average values of the quality characteristics are calculated. These averaged results are then tabulated in Table 5.

Expt	Tensile	S/N ratio for Tensile	Tensile Elongation	S/N ratio for
	Strength	Strength		Tensile
	(MPa)	(dB)		Elongation(dB)
1	155	44.504	5.42	14.6639
2	165	43.969	5.24	14.3534
3	171	45.090	5.37	14.5345
4	164	43.824	5.88	15.3580
5	172	45.499	5.76	15.2536
6	158	44.180	5.73	15.1782
7	169	45.062	6.34	16.0555
8	158	44.180	6.38	16.0828
9	166	45.8349	6.28	15.9730

Table 5:- Experimental results.

Selection Of Optimal Parameters

The experimental results, obtained from conducting the experiments, are shown in Table 5. The results illustrate the effect of the four control parameters on the two quality characteristics. Additionally, the Signal-to-Noise (S/N) ratios are presented in the same table. After performing the necessary calculations, graphs for each control parameter at its three levels of application are plotted. Figures 1 and 2 display these graphs.



Figure 1:- Main effects plot for S/N ratio for Tensile strength.

Figure 2:- Main effects plot for S/N ratio for tensile elongatin.



Tal	hle 6:-	Main	effect	for S/N	A ratio	for '	Tensile	strengt	h

Tuble 0 Main cheet for 5/10 faile for relisite strength.				
Level	A	В	С	
1	43.92	45.17	44.86	
2	43.92	45.29	45.30	
3	43.92	45.29	45.59	
Delta	0.059	0.13	0.73	
Rank	3	2	1	

Table 7:-	Main	effects	for	S/N	ratio	for	tensile e	elongation.
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Level	А	В	С
1	15.32	15.46	15.41
2	14.96	15.33	15.33
3	15.94	15.43	15.34
Delta	1.52	0.13	0.08
Rank	1	2	3

The delta values are calculated by determining the difference between the maximum and minimum S/N ratios for the corresponding levels of each parameter. These delta values help in assessing the significance of the control parameters in relation to each quality characteristic. The results obtained from the delta values are then compared with the data derived from performing Analysis of Variance (ANOVA) to further validate the findings. The optimal parametric setup for best Tensile strength is $A_2B_2C_3$ where $A_2 = 80$, B2 = 6 and $C_3 = 1100$.Similarly the optimal parametric setup for Tensile elongation is $A_3B_1C_1$ where $A_3 = 100$, B1 = 5 and $C_1 = 1100$

 Table 7:- Optimized levels for corresponding quality characteristic.

Process	Quality characteristic			
Parameter	Tensile	Strength	Tensile elongation	
	(Mpa)			

Α	80	100
В	6	5
С	1100	1100

Confirmation Tests

Confirmation tests were then conducted at the levels illustrated in Table 7 and the following results were obtained.

It is observed that with respect to the initial setup, performing the machining operations at the optimised setup gives us a better value for the quality characteristics.

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Performance Characteristic	Initial Setup	Optimised Setup	Initial value obtained	Optimised Value obtained
Tensile Strength	$A_1B_1C_1$	$A_2B_2C_3$	165	186
Tensile elongation	$A_1B_1C_1$	$A_3B_1C_1$	6.21	7.40

Theoretical And Experimental Comparison

The predicted optimum value of S/N ratios can be calculated from the following relationship

$$\eta_{\text{opt}} = \eta_{\text{m}} + \sum_{j=1}^{k} (\eta_{j} - \eta_{\text{m}}) ; j = 1....4$$
 (Eq.3)

Here

 $\eta_m = Grand mean of S/N ratio$

 \mathfrak{y}_i = Mean S/N ratio at optimum level

k = number of main design parameters that affect the quality characteristics.

Using the relationship provided in Eq. 3, the theoretical values for the S/N ratios of the quality characteristics are calculated at their optimal level of arrangement. These theoretical values are then compared with the experimental results obtained at the optimized levels, and the findings are tabulated in Table XI.

Table 9:- Comparison of the S/N ratios between experimental and Theoretical optimized results (in dB).

Quality characteristic	Experimental	Theoretical
TS	44.6599	44.6375
TE	16.1236	160006

The confirmation experiments show results that closely align with the theoretical values, indicating the accurac of the optimized parameter settings.

Anova For Single Level Optimization

ANOVA (Analysis of Variance) is a statistical decision-making tool used to detect differences in the average performance of groups of items being tested. It helps formally assess the significance of all main factors and their interactions by comparing the mean square of the factors against the estimate of error variance.

	L L				
Source	DOF	Adj SS	Adj MS	F-value	P- value
А	2	1.556	0.678	7	0.126
В	2	11.289	6.344	49	0.020
С	2	282.893	143.844	1274	0.001
Error	2	0.222	0.111		
Total	8	295.556			

Table 10:- ANOVA for Tensile strength.

Table 11:- ANOVA for Tensile elongation.

Source DOF Adi SS Adi MS F-value P-value
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А	2	1.55252	0.784211	139	0.008
В	2	0.01397	0.007478	1.259	0.444
С	2	0.00489	0.002444	0.349	0.739
Error	2	0.01119	0.006544		
Total	8	1.58136			

It was observed that, with respect to tensile strength, the most significant input parameters are tool rotation speed, axial force, and welding speed, with tool rotation speed being the most influential and welding speed being the least significant. In contrast, for tensile elongation, welding speed emerged as the most significant parameter, while tool rotation speed was found to be the least significant.

Multi Response Optimization

Taguchi method cannot be used directly to optimize the multi-response problems. The data Which was observed for each response using Taguchi designs can be analyzed by different methods to obtain a solution for a multi response problem. Here we attempt to perform a multi response optimization for all three quality characteristics taken all at once. The Taguchi

Grey relational analysis is used to perform this multi optimization. [13-21]

Grey based Taguchi method is a relatively new method proposed by J.Deng in 1982 for dealing poor, incomplete and uncertain systems [13]. In recent years, grey relational analysis becomes a powerful tool to analyses the processes with multiple performance characteristics for multiple engineering domains like rapid prototyping, wire EDM, welding etc. [14-17]. Strength of this method lies in the fact that it converts multiple responses into single response known as grey relational grade (GRG) which can be used for determining optimal factor setting for all the responses simultaneously [18]. Hence, Grey Taguchi method is used in this work to generate a single response from different performance characteristics. The multiple performance measures considered here are Tensile strength and tensile elongation of the tested samples. Both the responses need to be individually maximized whereas, overall grey relational grade, the multiple performance characteristic, is maximized. Grey relational analysis (GRA) is an impacting measurement method in grey theory that analyses uncertain relations among factors and interactions in a given system. It is actually a measurement of the absolute value of the data difference between sequences and it could be used to measure the approximate correlation between sequences.

The steps involved in the grey relational analysis are

- 1. Step 1- Convert the data obtained for the quality characteristics in terms of its S/N ratio (Y_{ij}) using the appropriate formula depending on the type of the quality characteristic.
- 2. Step 2-Normalize Y_{ij} as Z_{ij} ($0 \le Z_{IJ} \le 1$) to avoid the effect of using different units to reduce variability. Normalizing is a transformation performed on a single input to distribute the data evenly and scale it to acceptable range for further analysis.
- 3. Step 3- Compute the grey relational coefficient (GC) for the normalized S/N ratio values.
- 4. Step 4- Compute the grey relational grade (G_i)
- 5. Step 5- Use response graph method or ANOVA and select the optimal levels for the factors based on maximum average G_i values.

When the units in which performances are measured are different for different attributes, the influence of some attributes may be neglected. This may also happen if performance measures of some attributes have a large range. In addition, if the goals and directions of these attributes are different, this will cause incorrect results in the analysis. It is thus necessary that all attributes must have the same measurement scale. Therefore, normalization of data is done to process all performance values for every alternative into a comparability sequence. This process is called grey relational generation.

If the target value of original sequence is infinitely large then it has a characteristic of the "the-larger-the-better". The normalized experimental results for the larger the better characteristic can be expressed as:

$$Z_{ij} = \frac{Y_{ij} - \min(Y_{ij})}{\max(Y_{ij}) - \min(Y_{ij})}$$

This relation is used to normalize the values obtained for Tensile strength and tensile elongation.

When the target value of original sequence is infinitely small then it has a characteristic of the "the-smaller-the better". The normalized smaller the better characteristic is expressed as:

$$Z_{ij} = \frac{\max(Y_{ij}) - Y_{ij}}{\max(Y_{ij}) - \min(Y_{ij})}$$

After the grey relational procedure, all response values will be scaled into [0, 1]. An alternative will be the best choice if all of its performance values are closest to or equal to 1. However, this type of alternative does not usually exist. As a result reference sequence $Y_o = \{Y_{oj} = 1 | j = 1, 2, 3, ..., n\}$ is defined so as to find the alternative whose comparability sequence is the closest to the reference sequence. For this purpose, grey relational coefficient is calculated. Larger the grey relational coefficient, the closer are Y_{ij} and Y_{oj} . The grey relational coefficient can be expressed as:

$$GC_{ij} = \frac{\Delta_{\min} + \lambda \Delta_{\max}}{\Delta_{ij} + \lambda \Delta_{\max}}$$

Where

 Δ = absolute difference between Y_{oj} and Y_{ij} which is a deviation from the target value and can be treated as quality loss.

 Y_{oj} = Optimum performance value or the ideal normalized value of the jth response.

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 Y_{oj} = Optimum performance value or the ideal normalized value of the jth response.

Maximization is the goal in certain cases, and Grey Relational Analysis (GRA) is a powerful measurement method within grey theory that analyzes uncertain relationships among factors and interactions in a given system. It essentially measures the absolute value of the data differences between sequences and can be used to assess the approximate correlation between sequences.

The steps involved in the grey relational analysis are

- Step 1- Convert the data obtained for the quality characteristics in terms of its S/N ratio (Y_{ij}) using the appropriate formula depending on the type of the quality characteristic.
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- Step 5- Use response graph method or ANOVA and select the optimal levels for the factors based on maximum average G_i values.

When the units used to measure performance differ across attributes, the influence of certain attributes may be overlooked. This issue can arise if the performance measures for some attributes have a wide range. Additionally, if the goals and directions of these attributes vary, it can lead to incorrect results in the analysis. Therefore, it is essential that all attributes have the same measurement scale. To achieve this, data normalization is performed to transform all performance values for each alternative into a comparable sequence. This process is known as grey relational generation.

If the target value of the original sequence is infinitely large, it exhibits the characteristic of "the larger, the better." The normalized experimental results for the "larger-the-better" characteristic can be expressed as

$$Z_{ij} = \frac{Y_{ij} - \min(Y_{ij})}{\max(Y_{ij}) - \min(Y_{ij})}$$

This relation is used to normalize the values obtained for Tensile strength and tensile elongation.

When the target value of the original sequence is infinitely small, it exhibits the characteristic of "the smaller, the better." The normalized "smaller-the-better" characteristic is expressed as:

$$Z_{ij} = \frac{\max(Y_{ij}) - Y_{ij}}{\max(Y_{ij}) - \min(Y_{ij})}$$

After the grey relational procedure, all response values will be scaled into [0, 1]. An alternative will be the best choice if all of its performance values are closest to or equal to 1. However, this type of alternative does not usually exist. As a result reference sequence $Y_o = \{Y_{oj} = 1 | j = 1, 2, 3, ..., n\}$ is defined so as to find the alternative whose comparability sequence is the closest to the reference sequence. For this purpose, grey relational coefficient is calculated. Larger the grey relational coefficient, the closer are Y_{ij} and Y_{oj} . The grey relational coefficient can be expressed as:

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Where

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 Y_{oj} = Optimum performance value or the ideal normalized value of the jth response.

 Y_{ij} = The ith normalized value of the jth response variable

 $\Delta_{\min} =$ Minimum value of Δ

 $\Delta_{\rm max} =$ Maximum value of Δ

 λ is the distinguishing coefficient which is defined in the range $0 \le \lambda \le 1$. Here we have taken the value of λ to be 0.50 since we have two quality characteristics and we are giving the two of them equal weight age.

The purpose of the distinguishing coefficient is to expand or compress the range of grey relational coefficient. Whatever value of distinguishing coefficient is selected within the range of 0 and 1, they all lead to the same final design of factor levels [19-21].

The grey relational grade shows the level of correlation or degree of similarity between the comparability sequence and the reference sequence and is defined as:

$$G_i = \frac{1}{m} \sum GC_{ij}$$

The higher the value of the grey relational grade, closer is the corresponding factor combination to the optimal value.

Table	12:-	Gray	relational	grade.
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Experiment	Grey coefficient
1	0.425998
2	0.384716
3	0.598847
4	0.493092
5	0.845235
6	0.394377
7	0.928385
8	0.700010
9	0.690915

 Table 13:- Average grey grade for different levels.

Factor	Levels		
	1	2	3
А	0.476520	0.574235	0.763013
В	0.582492	0.629987	0.601380
С	0.496462	0.572908	0.747489

The grey grade is now treated as a single-response problem, and the data is analyzed to determine the optimal levels for the factors. By considering the average grey relational grades for the three input parameters at their respective levels, as shown in Table 13, it can be concluded that the multi-response optimization of the two quality characteristics occurs at the specified levels $A_3B_2C_3$.

Confirmation experiments for the above parametric setup for multi optimization have shown that the tensile strength and tensile elongation appear to be optimised.

Mathematical Models

Regression is performed on the data using MINITAB 17. The following equations is hence obtained for the 2 quality characteristics for single level optimization. They are:

Tensile Strength = 86.56 + 0.0167A + 1.167b + 0.06833C

Tensile elongation = 4.128 + 0.02542A - 0.0433B - 0.000083C

Where A, B, C are the process parameters

Conclusions:-

An attempt was made to optimize the FSW process with respect to the selected quality characteristics. The optimal levels for the process parameters were determined to achieve the best results for the quality characteristics.

- 1. For optimum Tensile strength, the recommended parametric combination is $A_2B_2C_3$ where A_2 is 80 mm/min, B_2 is 6kN and C_3 is 1100 RPM by using this optimal setup the Tensile strength was improved by 11.69%.
- 2. For optimum tensile elongation the recommended parametric combination is $A_3B_1C_1$ where A_3 is 100 mm/min, B_1 is 5kN and C_1 is 900 RPM. By using this optimal setup the tensile elongation was improved by 18.30%.
- 3. It is observed that the Tool rotation speed is the most significant parameter that affects the Tensile strength of Aluminium 6061 followed by axial force and welding speed.
- 4. Welding speed is the most significant parameter that affects the tensile elongation of Aluminium 6061 followed by axial force and tool rotation speed.
- 5. Multi optimization has been performed for both quality characteristics and the optimal level of the parametric arrangement is found to be $A_3B_2C_3$.
- 6. The mathematical models for the calculation of the quality characteristic (taken one at time) in terms of the process parameters have been obtained by regression.
- 7. By conducting the conformational experiments we can see that the results so obtained are in close line with the theoretical models that were obtained.

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