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RESEARCH ARTICLE

PROGRESSIVE ELEMENTS DETERMINANT.

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Abstract

It has been to construct a linear system from the equations, such that the coefficients of its variable constitute a finite progression when tabulated start from its first element a_{11} until its last element a_{nn}. The determinant, which includes those terms, called the progressive elements determinant. The finite progression that is mentioned above may be arithmetic progression, or geometric progression, and may be increasing or decreasing. In this paper the values of these determinants were deduced, and formulated ways to find their values.

Introduction:-

The linear system can be created, such that the variables coefficients constitute a square matrix[1],[2], and if the tabulated start from the first element a_{11} until the last element a_{nn} respectively, may be constituted *Arithmetic Progression* (*A.P.*), or *Geometric Progression* (*G.P.*). In general, the Arithmetic progression is a sequence of numbers such that the deference, *d*, between any term and its predecessor is the same, and the geometric progression is a sequence of numbers such that the ratio, *r*, of any term to its predecessor is the same [3], [4]. In any way, the (*A.P.*) and (*G.P.*) are finite progressions.

Consider the following linear system:

$$a_{11}\,x + a_{12}\,y + \cdots + a_{1n}\,z = c_1$$
$$a_{21}\,x + a_{22}\,y + \cdots + a_{2n}\,z = c_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad = \vdots$$
$$a_{n1}\,x + a_{n2}\,y + \cdots + a_{nn}\,z = c_n$$

$c_1, c_2, \dots, c_n \neq 0$. It can be solved by Cramer's rule.

Since $a_{11}, a_{12}, \dots, a_{1n}, a_{21}, a_{22}, \dots a_{2n}, \dots, a_{nn}$ is (*A.P.*) or (*G.P.*), and has common ratio, or common difference. The order (*n*) of the determinant is split the terms of the progression into *n* rows and *n* column, each one of them containing *n* terms. The elements of the determinant mentioned are read a row after a row. It is valuable to understand that the progression (*A.P.*) or (*G.P.*) in determinant are classified into rows, this act allows for the generalizations about the paper subject.

Definitions And Basics

Definition (2.1):

The Progression Elements Determinant,(PED), is the determinant whose elements are starting from the first element until the last element constituted Arithmetic progression or Geometric Progression, increasing or decreasing.

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Formulas of the Progression Elements Determinant:

The subject of this paper confounds two topics: determinants and progressions, so we need to formation formulas and symbols on different *PED*s. Consider that $n \times n$ is order of *PED*, *N* is the terms number of the progression (elements number of *PED*), such that $N = 1,2,\dots,k$, and $N,k \in N^+$ are finite numbers; so one can adopt the following formulas:

i) The *PED* with Arithmetic Progression denoted by $PED_{(A.P.)}$, so its general form given by:

$$\left|PED_{(A.P.)_{n\times n}}\right| = |s + (N-1)d| \dots\dots (1)$$

Such that s the first element of the *PDF*, i.e. the first term of (*A.P.*), and $d \in R$ is the Common Difference.

ii) The *PED* with Geometric Progression denoted by $PED_{(G.P.)}$, so its general form given by:

$$\left|PED_{(G.P.)_{n\times n}}\right| = |s.r^{N-1}| \dots\dots (2)$$

Such that s the first element of the *PDF*, i.e. the first term of (*G.P.*), and $r \in R$ is the Common Ratio.

iii) The value of any *PED* given by $D|\,PED|$.

iv) And for any *PED*:

$$N = n^2 \dots\dots (3)$$

The Row (Column) Progressions

Let [*PED*] be a square matrix, $[PED]^t$ its transpose, then:

$$D|PED| = D|PED|^t \dots\dots (4)$$

Certainly, that obtaining of the $|PED|^t$ will change the fashion progression terms by interchange rows and the corresponding columns. This change causes the progression to be cut off, Thus progressions are created in each row or in each column, which are not interrelated, but the common deference (d_{common}) or common ratio(r_{common}), will keep its value. So we can indicate to this kind of *PED* by:

$$PED_{row(column)} \dots\dots (5)$$

That is tell us the progressions are separated .i.e. they not consist from its first element a_{11} until its last element a_{nn}. The formula (4) is a generalization of the following two formulas:

$$PED_{row} \dots\dots (6),$$

with d_{common} or with r_{common} for every row in *PED*. And

$$PED_{column} \dots\dots (7),$$

with d_{common}, or with r_{common} for every column in *PED*.

The $PED_{row(column)}$ may be $|PED|^t$ or not, so the regularity of the all determinant elements without cutting the gradation is not necessary condition to construct a *PED*. To distinguish the two types of progressions that are mentioned in formulas (6) and (7), we called them: *row progressions*, and *column progressions*, respectively. One can refer to them in more detail as in the following formulas:

$$\left.\begin{matrix} PED_{row(column)(A.P.)} \\ PED_{row(column)(G.P.)} \end{matrix}\right\} \dots\dots (8)$$

Definition:

The *Row Progressions* are collection of progressions elements determinant, each one of them exists in a row of the determinant with common deference (ratio). However, the deference (ratio) between last term of row_i progression and first term of the next progression row_{i+1} is not same.

The terms of the Row Progressions are given by: row_1progression, row_2progression, …, row_nprogression. So, the elements of $PED_{row(A.P.)}$ given by:

$$PED_{row(A.P.)} = s_{i1} + (N-1)d \dots\dots (9)$$

Since $i = 1,2,\dots,n$. n is order of the determinant.

In addition, the elements of $PED_{row(G.P.)}$ denoted by:

$$PED_{row(G.P.)} = s_{i1}r^{N-1} \dots\dots\dots\dots (10)$$

Definition:

The column Progressions are collection of progressions elements determinant, each one of them exists in a column of the determinant with common deference (ratio). However, the deference (ratio) between last term of column_{i} progression and first term of the next progression$\text{column}_{\text{i+1}}$ progression is not same. The terms of the Row Progressions are given by: $colmun_1 progressions, column_2\, progressions, \dots, column_n progressions$.

So, the elements of $PED_{column(A.P.)}$ denoted by:

$$PED_{column(A.P.)} = s_{1j} + (N-1)d \ \ldots\ldots (11)$$

Since $j = 1,2,\dots,n$ is number of column, n is order of the determinant.

In addition, the elements of $PED_{column(G.P.)}$ are denoted by:

$$PED_{column(G.P.)} = s_{1j} r^{N-1} \ldots\ldots (12)$$

Since *i* and *j* are as in the formulas (9), (10), and(11).

Evaluation Of Progressive Elements Determinant

Because of the different types of *PED*, the rules are needed to evalute them, and covering these types.

Rule

If the order of *PED* is $n = 2$ then:

i) $D\left|PED\ {}_{(A.P.)}{}_{2\times2}\right| = -2d^2$, $d \in R$ is the common difference.

ii) $D\left|PED_{(G.P.)}{}_{2\times2}\right| = 0$.

Proof: *i*) if *N* is increasing:

$$D\left|PED_{(A.P.)}{}_{2\times2}\right| = D|s + (N-1)d| = \begin{vmatrix} s & s+d \\ s+2d & s+3d \end{vmatrix} = -2d^2.$$

and if *N* is decreasing, it is clear that:

$$D\left|PED_{(A.P.)}{}_{2\times2}\right| = D|s + (N-1)d| = \begin{vmatrix} s+3d & s+2d \\ s+d & s \end{vmatrix} = -2d^2$$

Proof: *ii*) if *N* is increasing:

$$D\left|PED_{(G.P.)}{}_{2\times2}\right| = D|s.r^{N-1}| = \begin{vmatrix} s & sr \\ sr^2 & sr^3 \end{vmatrix} = 0.$$

and if *N* is decreasing, it is clear that:

$$D\left|PED_{(G.P.)}{}_{2\times2}\right| = D|s.r^{N-1}| = \begin{vmatrix} sr^3 & sr^2 \\ sr & s \end{vmatrix} = 0.$$

Rule

If the order of *PED* is $n \geq 3$, n finite set, then the $PED_{(G.P.)}$ or $PED_{(A.P.)}$ equal to, zero.

Proof: The *Laplace Expansion* for any determinant is given by the following formula:

$det(A) = \sum_{k=1}^{n} (-1)^{i+j}\, a_{ik}\, A_{ik}\, (-1)^{i+j}\, a_{ik}\, A_{ik}$, $1 \leq i \leq n$ [5],[6]. Depending on this formula can be written as the expansion of $PED_{(A.P.)}$ as follows:

$$D|s + (N-1)d| = (-1)^{i+j}[s + (n-1)d]|s + (N-1)d|_{(i-1)\times(k-1)} \ldots (13)$$

And the expansion of $PED_{(G.P.)}$ as follows:

$$D|s.r^{n-1}| = (-1)^{i+k}(s.r^{n-1})|s.r^{n-1}|_{(i-1)\times(k-1)} \ldots (14)$$

Such that s is the first element in $PED_{(G.P.)}$ and in $PED_{(A.P.)}$.

Given that $n \geq 3$ for order of *PED* require using the *mathematical induction* to prove this rule.

a) To prove that $D\left|PED_{(A.P.)}{}_{n\times n}\right| = 0$, $n \geq 3$, by using first row to find expansion of the determinant:

Step 1. $n = 3$, then:

$$D\left|PED_{(A.P.)}{}_{3\times3}\right| = \begin{vmatrix} s & s+d & s+2d \\ s+3d & s+4d & s+5d \\ s+6d & s+7d & s+8d \end{vmatrix}$$

$= -6sd^2 + 6sd^2 + 6d^2 - 6d^2$
$= 0$

Step2. $n > 3, n < k$, are finite sets, then:

$D\left|PED_{(A.P.)}{}_{n>3}\right|$
$= [s + (1-1)d](-1)^{1+1}|s + (N-1)d|_{1\times(n-1)} + [s$
$+(2-1)d(-1)^{1+2}\,|s + (N-1)d|_{(n-1)\times(n-2)} + \cdots$
$+[s + (N-2)d](-1)^{1+(k+2)}|s + (N-1)d|_{1\times(k-2)} + [s$
$+\{N-(k-1)\}(-1)^{1+(k-1)}|s + (N-1)d|_{1\times(k-1)} + \cdots$
$+[s + (N-1)d](-1)^{1+k}|s + (N-1)d|_{1\times k}$
$= 0$

So, $PED_{(A.P.)} = 0$ for the order $n \geq 3$.

By the same way can prove the rule (3.3) when N is decreasing.

b) To prove that $D\left|PED_{(G.P.)}{}_{n\times n}\right| = 0$, n ≥ 3 , by using first row to find expansion of the determinant:

Step1. $n = 3$, then:

$D\left|PED_{(G.P.)}{}_{3\times3}\right| = D|s.r^{n-1}|$
$= s[sr^4 \cdot sr^8 - sr^5 \cdot sr^7] - sr[sr^3 \cdot sr^8 - sr^5 \cdot sr^6]$
$+sr^2[sr^3 \cdot sr^7 - sr^4 \cdot sr^6]$
$= 0$

Step2. $n > 3, n < k$, are finite sets, then:

$D\left|PED_{(G.P.)}{}_{n>3}\right| = D|s.r^{N-1}|$
$= sr^{1-1}(-1)^{1+1}|s.r^{n-1}|_{1\times(n-1)} + sr^{2-1}(-1)^{1+2}|s.r^{n-1}|_{1\times(n-2)}$
$+ \cdots + sr^k(-1)^{1+k}|s.r^{n-1}|_{1\times k}$
$= 0$

So, $PED_{(G.P.)} = 0$, for the order $n \geq 3$.

By the same way, can prove the rule (4.2) when N is decreasing.

Independent on the rules (4.1), and (4.2) that are mentioned above, one can write the following rule.

Rule

If $n \geq 3$ is the order of determinant, n is finite set, then $D\left|PED_{row(column)}\right| = 0$.

Proof: by proof method of rule (4.2).

Rule

If the order of *PDF* is $n \geq 3$, then $D|PED_{row}|^t = D|PED_{column}|$, and $D|PED_{column}|^t = D|PED_{row}|$.

Proof: The common deference or common ratio in PED_{row} and in PED_{column} is the same value. And by using the Laplace Expansion can complete the prove.

Rule

If $n = 2$ is the order of $PED_{(G.P.)}$, then:

i) $D|PED_{row(G.P.)}|^t = D\left|PED_{column(G.P.)}\right|$, and

ii) $D|PED_{column(G.P.)}|^t = D\left|PED_{row(G.P.)}\right|$,

provided that they have same common ratio.

Proof: it is clear.

Examples

the following examples are explaining and applying the mentioned rules:

a) $A = \begin{vmatrix} 20 & 100 & 0 \\ 22 & 102 & 2 \\ 24 & 104 & 4 \end{vmatrix}$ is $PED_{column(A.P.)}$, $d_{common} = 2$, $det(A) = 0$, and

$A^t = \begin{vmatrix} 20 & 22 & 24 \\ 100 & 102 & 104 \\ 0 & 2 & 4 \end{vmatrix}$ is $PED_{row(A.P.)}$, $d_{common} = 2$,$det(A^t) = 0$.

b) $B = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is $PED_{row(A.P.)}$, $d_{common} = 1$, $det(B) = 0$,and

$B^t = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$ is $PED_{column(A.P.)}$, $d_{common} = 1$, $det(B^t) = 0$.

c) $C = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 4 & 12 & 8 \end{vmatrix}$ is $PED_{column(G.P.)}$, $r_{common} = 2$, $det(C) = 0$, and

$C^t = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{vmatrix}$ is $PED_{row(G.P.)}$, $r_{common} = 2$, $det(c^t) = 0$.

d) $D = \begin{vmatrix} 2 & 4 \\ 16 & 32 \end{vmatrix}$ is $PED_{row(G.P.)}$, $r_{common} = 2$, $det(D) = 0$, and $D^t = \begin{vmatrix} 2 & 16 \\ 4 & 32 \end{vmatrix}$ is $PED_{column(G.P)}$, $r_{common} = 2$, $det(D^t) = 0$. And the convers is true. On other hand may be $D = \begin{vmatrix} 2 & 16 \\ 4 & 32 \end{vmatrix}$ is $PED_{row(G.P.)}$, $r_{common} = 8$, $det(D) = 0$, and $D^t = \begin{vmatrix} 2 & 4 \\ 16 & 32 \end{vmatrix}$ is $PED_{column(G.P.)}$, $r_{common} = 8$, $det(D^t) = 0$. And so on.

Comment:

If $n = 2$ is the order of PED, then $D\left|PED_{row(column)(A.P.)}\right| \neq -2d^2$. For example the following $PED_{row(column)(A.P.)}$: $Z = \begin{vmatrix} 1 & 10 \\ 2 & 11 \end{vmatrix} = -9$, whereas the $d_{common} = 9$ for $PED_{row(A.P.)}$, and the $d_{common} = 1$ for $PED_{column(A.P.)}$. This exemption requires interpretation. The cause reasonable is the terms in $PED_{row(column)(A.P.)}$, when its order $n = 2$ are not as in the row(column) geometric progression, for this reason, the mentioned exemption is not subject to Rule (4.1), although ${PED^t}_{row(column)(A.P.)} = PED_{column(row)(A.P.)}$, $n = 2$, and conversely.

The Results:-

Instead of the Laplace Expansion method to find the values of the Progressive Elements Determinant can use the following properties, which are added to the known properties of the determinants. Let's take that n is set of finite natural numbers:

1. The determinant with order $n = 2$, and its elements constitute arithmetic progression start from the first element a_{11} until the last element a_{nn} equal to, $-2d^2$, such that *d* is the common deference.
2. The determinant with order $n = 2$, and its elements constitute geometric progression start from the first element a_{11} until the last element a_{nn} , row after row, equal to, zero.
3. The determinant with order $n \geq 3$, and its elements constitute arithmetic (geometric) progression start from the first element a_{11} until the last element a_{nn} equal to, zero.
4. The transpose of determinant with order $n = 2$, and its elements in any row constitute row geometric progression, equal to the determinant which its column constitute columns geometric progression, with same ratio, and each one of the determinants that are mentioned is equal to zero.
5. The determinant with order $n \geq 3$, and its elements in any row(column) constitute row(column) arithmetic (geometric) progression, equal to zero.

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