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RESEARCH ARTICLE

IDI-75 - AN OPTIMAL ONES ASSIGNMENT METHOD FOR SOLVING UNBALANCED ASSIGNMENT PROBLEMS

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Abstract

In this article, a new technique namely IDI-75 (named to commemorate the **75**th **I**ndependence **D**ay of **I**ndia), which is a simplified form of the existing Ones Assignment Method (OAM) of solving assignment problems, finds the optimal solution to a given unbalanced assignment problem (UAP). In the OAM, it is required to convert the given UAP into a balanced one by introducing one dummy row or column with 1 unit of effectiveness in each cell. But, the IDI-75 technique can be applied directly on the given UAP without converting it into a balanced one. The technique requires only row minimum division operation or only column minimum division operation depending on the size of rows and columns in order to have at least one 1-entry in each of the required number of rows and/or columns only. The assignments are made on the appropriate 1-entry cells of reduced ratio of costs matrix. To test the validity and effectiveness of the IDI-75 technique, 20 benchmark instances with different sizes from the literatures have been tested. Simulation results authenticate that the technique IDI-75 is the best one which produces optimal solution to all 20 instances. Therefore, it is clever to apply the IDI-75 technique to solve the UAPs as it is very simple, easy to understand, easy to apply and consume less time in comparison to the existing OAM.

Introduction:-

The assignment problem is one of the most essential applications in the real world and it is a special class of linear programming in which the objective is to assign n number of jobs to n number of persons or machines at an overall minimum cost or maximum profit. Assignment may be jobs to persons, operators to machines, drivers to trucks, trucks to delivery routes, classes to rooms, or problems to research teams, etc. There are various ways to solve the unbalanced (and balanced) AP. A well known iterative solution procedure, based on zeros assignment, is developed by H.W. Kuhn [2] in 1955 named as Hungarian method. In the recent years considerable numbers of methods have been published by several researchers to find the optimal solutions for unbalanced (and balanced) APs. But, these methods were not able to produce optimal assignment plans to all APs. We briefly make a glance into the recent methods developed for solving APs.

In 2012, Hadi Basirzadeh [1] introduced a new approach to APs namely, Ones Assignment Method (OAM) for solving a wide range of such problems. In the process of this method, the author first defines the assignment matrix,

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then by using determinant representation he obtains a reduced matrix which has at least one 1 in each row and column. Then by using the OAM, he obtains an optimal solution for AP by assigning 1s to each row and each column. This method can be applied only when all the entries of the cost matrix are nonzero. To convert an UAP into BAP, the author adds one dummy row or column which all elements are one.

In 2013, Ghadle K.P. and Muley Y.M. [3] presented a new method namely, Revised Ones Assignment (ROA) method for solving a wide range of APs, which is different from the OAM. This method is also based on creating some 1s in the assignment matrix and then tries to find a complete assignment in terms of 1s.

In February 2014, M.D.H. Gamal [4] had brought out some drawbacks in the OAM due to Hadi Basirzadeh [1], and gave a remedial strategy to overcome the case when some entries of the cost matrix are zeros. The author also gave examples of the APs where the OAM fails to find their optimal solution.

In May 2014, M. Khalid et al. [5] introduced the New Improved Ones Assignment (NIOA) method, which leads to brief computation time comparatively and will attain an exact optimal solution. Besides, this improved version will overcome the drawbacks as indicated by M.D.H. Gamal [4].

In December 2020, R. Murugesan and T. Esakkiammal [6] revealed that the ROA method as well as the NIOA method for solving APs do not present optimal solution at all times. The authors also gave examples of the APs where the ROA and the NIOA methods fail to find their optimal solution.

In January 2021, R. Murugesan and T. Esakkiammal [7] introduced a new technique namely 'Mantra', which is a simplified form of the existing Hungarian method, finds optimal solution directly to any given UAP. In the Hungarian method of solving, it is required to convert the given UAP into a balanced one by introducing dummy row(s) or column(s) having zero effectiveness in each cell. But, the 'Mantra' technique can be applied directly on the given UAP without converting it into a balanced one.

In April 2021, T. Esakkiammal and R. Murugesan [10] introduced a new ones assignment method namely 'Mass' for finding optimal pattern of assignments to a wide range of APs. In this method, suppose the AP is unbalanced, for conversion to balanced required number of additional rows or columns to the assignment cost matrix to equalize with the columns or rows is introduced. The assignment cost for each cell in these dummy rows or columns is set to 1.

In May 2022, R. Murugesan [8] projected a new technique named CASSI to obtain an optimal solution to APs. This method finds the optimal solution to the given AP in two phases. The added advantage of this technique is that for any AP, the solution obtained by applying any method based on 'zeros assignment approach' can be tested for optimality and can also be improved towards optimal, if it's not optimal.

In the same period May 2022, R. Murugesan [9] proposed a very simple and innovative method named E-SOFT to find the optimal assignment plans to the UAPs. It is an alternative simple method to solve UAPs in addition to the 'Mantra' technique.

In this paper, we have proposed a new ones assignment technique, namely IDI-75, to find the optimal pattern of assignments to UAPs. Like the 'Mass' method, it is not required to convert the given UAP into a balanced one by introducing dummy row(s) or column(s) having effectiveness 1 in each cell. The technique can be applied directly on the given UAP. The IDI-75 technique has been tested for 20 benchmark problems from the literatures and obtained optimal solution to each problem.

The paper is organized as follows: In Section 1, brief introduction about the recent methods developed for solving the APs is given. In Section 2, the algorithm of the existing 'Mantra' technique is presented. The algorithm for the proposed IDI-75 technique is presented in Section 3. In Section 4, one benchmark UAP from the literature has been illustrated. Section 5 lists a set of 20 benchmark UAPs from the literatures, which are all tested by the IDI-75 technique. The results produced by the IDI-75 technique are compared with the results of the 'Mnatra' technique and are shown in Section 6. The differences between the IDI-75 and 'Manta' techniques are listed in Section 7. Finally, in Section 8 conclusions are drawn.

2. Algorithm for the Existing Mantra Technique

The readers are requested to refer [7] for the algorithm and explanation of the 'Mantra' technique due to R. Murugesan and T. Esakkiammal.

3. Algorithm for the Proposed IDI-75 Assignment Technique

The term IDI-75 has been derived from the phrase '**75**th **I**ndependence **D**ay of **I**ndia'. Commemorating and celebrating the 75th Independence Day of India on 15th August 2022 is a significant event as it stands as a remainder of the sacrifices that many great known and unknown freedom fighters made to get independence from the 200-year British rule on 15th August 1947. Usually it is observed throughout the Indian nation with the hoisting of the tricolor Indian National Flag, parades, and colorful events. As citizen of our nation, we have developed a technique and named it as IDI-75 to commemorate the historical day. The IDI-75 technique consists of two phases. In the first phase, a complete assignment plan is found out using the ones assignment technique based on the ME rules and in the second phase optimality testing and optimizing of the obtained complete assignment plan is carried out based on the computed improvement indices of the unassigned cells. The algorithm is as follows:

We use the following abbreviations and notations followed by two operations defined for the development of the algorithm:

OAC – Original Assignment Cost
RMD – Row Minimum Division
CMD – Column Minimum Division
RCM – Ratio of Costs Matrix
CAP – Complete Assignment Plan
OAP – Optimal Assignment Plan
MAP – Modified Assignment Plan
NCC – Net Cost Change
Z – Overall Assignment Cost
Z^* – Minimum Overall Assignment Cost

The RMD Operation

Divide each of the costs of every row of the given UAP by its minimum cost. This will result in a ratio matrix, which will have at least one 1s in each row.

The CMD Operation

Divide each of the costs of every column of the given UAP by its minimum cost. This will result in a ratio matrix, which will have at least one 1s in each column.

Phase-I (Finding a CAP)

Step 1: Conversion into Minimization UAP.

If the given UAP is of maximization type, then convert it into a minimization one.

Step 2: Check the Non-zero Entry.

If all the entries of the given assignment cost matrix are nonzero, go to Step 3. Otherwise, add 1 with every entry of the row(s) having zero cost entry. This will ensure that all the entries of the cost matrix are nonzero. Go to Step 3.

Step 3: Find the minimum of number of rows and columns of the cost matrix and decide the operation.

If $m \times n$ is the size of the given cost matrix, find the minimum of m and n, in brief Min{m, n}, where m and n denote the number of rows and columns respectively of the cost matrix. Let Min{m, n} = k

a) If $m < n$, then perform the Row Minimum Division (RMD) operation and go to Step 4 for making the assignments.

b) If $n < m$, then perform the Column Minimum Division (CMD) operation and go to Step 4 for making the assignments.

Note: The resultant matrix obtained in Step 3(a) or Step 3(b) is known as the 'ratio of costs matrix' (RCM). It is noted that there will be at least one 1-entry in each row when $m < n$ and in each column when $n < m$ of an RCM. The cells having only 1-entry in an RCM are called '1-entry cells'.

Step 4: Cover all the 1s with minimum number of lines using ME rules and obtain a CAP (Refer Appendix A)
Note: There may be three types of cells in the assignment matrix obtained via Step 4 namely, 'the assigned 1-entry cells', 'the unassigned 1-entry cells' and 'the unassigned >1 entry cells'. The assigned 1-entry cells are called 'assigned cells' or 'occupied cells' or 'used cells' and the remaining cells are called 'unassigned cells' or 'unoccupied cells' or 'unused cells' or 'empty cells'.

Phase-II (Optimality testing and optimizing the obtained CAP)
Step 1: **Compute the I-index for every unused cell**
In the assignment table with the 'original assignment costs' (OACs), trace a loop starting and ending at an unused cell. There may be no loop or one loop or more than one loop from an unused cell. Mark (+) and (-) sign alternatively at each corner of a loop, starting from the unused cell. Compute the effect on cost for the selected unused cell, by adding together the OACs found in each cell containing a plus sign and then subtracting the OACs found in each cell containing a minus sign. This effect on cost is called the '**net cost change'** (NCC) value for the unused cell. If the unused cell has more than one loop, then compute the NCC value for the cell associated with each loop. The maximum among them is considered as the '**Improvement index'** (or simply '**I-index'**) of the unused cell. In the same way, compute the I-index for every unused cell in the assignment matrix.

Note: I-index for an unused cell may be negative or zero or positive. If we make a new assignment only in the unused cell with negative I-index, then the 'overall assignment cost' Z may decrease. Do not select the > 1 entry unused cell for new assignment, if a loop cannot be traced from it. However, select the 1-entry cell for new assignment, if a loop cannot be traced from it.

Step 2: **Test the optimality condition**
If the I-index for each unused cell is non-negative, then definitely the current CAP is an optimal one for the given UAP. If negative I-index occurs for certain unused cells, the current CAP is not optimal and it has to be improved further.

Step 3: **Optimize the current CAP**

(i) *Select an appropriate unused cell for new assignment*. Select the unused cell with the largest negative I-index (-1, -2, -3 means select -3) to include in the new CAP. If tie occurs among the unused cells with identical largest negative I-index, then select each such cell for the new assignment as a separate case. Such a situation may generate an alternative 'optimal assignment plan' (OAP) to the given UAP. The largest negative I-index of an unused cell indicates the cost decrease that can be achieved by making an assignment in that cell.

(ii) *Make a new assignment in the selected cell*. If the cell (i, k) has the largest negative I-index in the i^{th} row and the cell (i, j) is the currently assigned cell in the i^{th} row, move the assignment from the cell (i, j) to the new cell (i, k). Equivalently, the assignment in the j^{th} column is first moved to the k^{th} column. Due to the unique assignment property in a row and column, this move will induce the current assignment in the k^{th} column, say (m, k) to move to another appropriate column. So, move the allocation from the cell (m. k) to the cell (m, n) having largest negative I-index or next to the largest negative I-index. Move the current assignments in this way until to get a new assignment in the j^{th} column from which we have started our first move. Due to these moves a 1-entry cell (from which a loop cannot be traced) and a > 1-entry cell may also get an assignment.

(iii) *Write the modified assignment plan*. Write the corresponding 'modified assignment plan' (MAP) and compute the associated 'overall assignment cost' Z.

Step 4: R**epeat the process**
Repeat the Steps 1 to 3 until there is no negative I-index for all unused cells or there is no further reduction in Z of a CAP. That is, the CAP is an optimal one. Write the 'optimal assignment plan' (OAP) and compute the associated 'minimum overall assignment cost' (Z^*).

Unique/Alternative OAP
In an optimal assignment table, if an unused cell has I-index zero, it indicates that the given UAP will have an alternative OAP. Also, if the I-indices for all the unused cells are strictly > 0, then the given UAP has a unique OAP only.

4. Numerical Illustrations

Appropriate illustrative explanation makes the readers to understand the proposed IDI-75 technique systematically. Bearing in mind, one UAP from the literature has been illustrated.

Example: Consider the following cost minimizing UAP with three jobs and four machines, which is shown in Table 1.

Table 1:- The given Minimization UAP.

	Machines			
Jobs	**1**	**2**	**3**	**4**
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

Solution by the proposed 'IDI-75' technique

Phase-I: (Finding a CAP).

By applying the steps of Phase-I in the IDI-75 technique, one can get the assignment matrix with a CAP, as shown in Table 2. The cells with the starred 1s denote the assigned cells.

Table 2:- Ratio of costs matrix with CAP.

	Machines			
Jobs	**1**	**2**	**3**	**4**
1	1	1	**1***	1.78
2	**1***	1.22	1.36	2.37
3	1	**1***	1.22	2.20

The obtained CAP is (1, 3), (2, 1) and (3, 2) with Z = 28+8+15 = $51.

Phase-II: (Optimality testing and optimizing the obtained CAP)

In order to perform the optimality test and optimize the obtained CAP, consider the assignment table with the OACs, as shown in Table 3.

Table 3:- The assignment table with the OACs.

	Machines			
Jobs	**1**	**2**	**3**	**4**
1	1 18	1 24	**1*** 28	1.78 32
2	**1*** 8	1.22 13	1.36 17	2.37 19
3	1 10	**1*** 15	1.22 19	2.20 22

First iteration

Step 1: Computing the I-index for every unused cell.

The computation of NCC values and hence the I-indices for the unused cells in the assignment Table 3 are shown in Table 4.

Table 4:- The I-indices for the unused cells found in Table 3.

Unused Cells	NCC value(s) due to the possible loops traced	I-index	Type of cell
(1, 1)	18-10+15-24 = -1	-1	1-entry cell
(1, 2)	24-18+10-15 = 1	1	1-entry cell
(1, 4)	No loop	----	
(2, 2)	13-8+18-24 = -1	-1	>1 entry cell
(2, 3)	17-8+18-28 = -1	-1	>1 entry cell
(2, 4)	No loop	----	

(3, 1)	10-18+24-15 = 1	1	1-entry cell
(3, 3)	19-28+24-15 = 0 19-28+18-10 = -1	 0	>1 entry cell
(3, 4)	No loop	----	

Step 2: Testing the optimality condition
As negative I-index occurs for certain unused cells, the current CAP is not optimal and it has to be improved further.

Step 3: Optimizing the current CAP
The largest negative I-index is -1 which corresponds to the unused cells (1, 1), (2, 2) and (2, 3). If one makes assignment in these cells, the overall assignment cost Z will be reduced further. We try a new assignment in these cells one by one, each as a separate case.

Case (1): Select the cell (1, 1) for new assignment.
First, a new assignment is placed at the cell (1, 1). Due to this, the induced modified assignments are shown in Table 5.

Table 5:- Modified assignments due to new assignment at (1, 1).

Currently assigned cell	Newly assigned cell	I-index
(1, 3)	(1, 1)	-1
(2, 1)	(2, 2)	-1
(3, 2)	(3, 3)	0
	Overall I-index	-2

As the 'overall I-index' for the modified assignments is negative, definitely there will be a reduction in Z. The 'modified assignment plan' (MAP) is (1, 1), (2, 2) and (3, 3) with the overall cost Z = 18+13+19 = \$50. Note that, due to the new assignment at the cell (1, 1) the Z value has been reduced by \$1.

Case (2): Select the cell (2, 2) for new assignment.
Next, a new assignment is positioned at the cell (2, 2). Due to this, the induced modified assignments are shown in Table 6.

Table 6:- Modified assignments due to new assignment at (2, 2).

Currently assigned cell	Newly assigned cell	I-index
(2, 1)	(2, 2)	-1
(3, 2)	(3, 3)	0
(1, 3)	(1, 1)	-1
	Overall I-index	-2

As the 'overall I-index' for the modified assignments is negative, definitely there will be a reduction in Z. The MAP is (1, 1), (2, 2) and (3, 3) with Z = 18+13+19 = \$50. Observe that, the new assignment assigned at the cell (2, 2) have generated the same identical MAP, as obtained in case (1).

Case (3): Select the cell (2, 3) for new assignment.
Next, a new assignment is located at the cell (2, 3). Due to this, the induced modified assignments are shown in Table 7.

Table 7:- Modified assignments due to new assignment at (2, 3).

Currently assigned cell	Newly assigned cell	I-index
(2, 1)	(2, 3)	-1
(1, 3)	(1, 1)	-1
	Overall I-index	-2

As the ‘overall I-index’ for the modified assignments is negative, definitely there will be a reduction in Z. The MAP is (1, 1), (2, 3) and (3, 2) with Z = 18+17+15 = \$50. Observe that, due to the new assignment at the cell (2, 3) the Z value has been reduced by \$1. Also observe that, this obtained MAP is different, compared to that of obtained in case (1) or case (2).

Second iteration
Consider the modified assignment table obtained from case (1) with the OACs, as shown in Table 8. The starred cells are the assigned cells.

Table 8:- The modified assignment table with the OACs.

	Machines			
Jobs	**1**	**2**	**3**	**4**
1	**1*** 18	1 24	1 28	1.78 32
2	1 8	**1.22 *** 13	1.36 17	2.37 19
3	1 10	1 15	**1.22*** 19	2.20 22

Step 1: Computing the I-index for every unused cell.
The computation of NCC values for the unused cells found in the assignment Table 8 are shown in Table 9.

Table 9:- The I-indices for the unused cells found in Table 8.

Unused Cells	NCC value(s) due to the possible loops traced	I-index	Type of cell
(1, 2)	24-13+8-18 = 1	1	1-entry cell
(1, 3)	28-19+15-24 = 0		1-entry cell
	28-19+10-18 = 1	1	
(1, 4)	No loop	----	
(2, 1)	8-13+24-18 = 1		1-entry cell
	8-13+15-10 = 0	1	
(2, 3)	17-28+24-13 = 0		
	17-19+15-13 = 0	**0**	
(2, 4)	No loop	----	
(3, 1)	10-8+13-15 = 0		1-entry cell
	10-18+24-15 = 1		
	10-19+28-18 = 1		
	10-19+28-24+13-8 = 0	1	
(3, 2)	15-13+8-10 = 0		1-entry cell
	15-24+18-10 = 1		
	15-19+28-24 = 0		
	15-19+28-18+8-13 = 1	1	
(3, 4)	No loop	----	

Step 2: Testing the optimality condition
As the given UAP is minimization one and all the I-indices for all the unused cells are nonnegative, the current CAP (1, 1), (2, 2), (3, 3) is optimal. Also, as the I-index for the unused cell (2, 3) is 0, the given UAP has alternative OAP. If we make an assignment at the cell (2, 3) we get the alternative OAP.

Write the OAPs.

The two OAPs are (1, 1), (2, 2), (3, 3) and (1, 1), (2, 3) and (3, 2) each with $Z^* = \$50$.

Solution by the MANTRA technique
If we solve the given UAP, shown in Table 1, by applying the steps of the ‘Manta’ technique, the two obtained OAPs will be

(i) (1, 1), (2, 2), (3, 3) with $Z^* = \$50$.
(ii) (1, 1), (2, 3), (3, 2) with $Z^* = \$50$.

Important Remark
From the two assignment solution procedures, it is ascertained that the 'IDI-75' technique as well as the 'Mantra' technique have produced the same number and the same identical OAPs to the given UAP.

Numerical Examples
To justify the efficiency of the proposed IDI-75 assignment technique, we have solved 20 numbers of benchmark UAPs of unbalanced category in different sizes, from various literatures and books, which are listed in Table 10.

Table 10:- List of benchmark UAPs for testing.

Problem No.,(Author(s), Year,)	Problem No.,(Author(s), Year)
***Problem 1** $[C_{ij}]$ 3×4= [18 24 28 32; 8 13 17 19; 10 15 19 22]	***Problem 11** $[C_{ij}]$ 6×4= [6 5 1 6; 2 5 3 7; 3 7 2 8; 7 7 5 9; 12 8 8 6; 6 9 5 10]
Problem 2 $[C_{ij}]$ 4 ×3= [10 7 8; 8 9 7; 7 12 6; 10 10 8]	***Problem 12** $[C_{ij}]$ 6×5= [6 2 5 2 6; 2 5 8 7 7; 7 8 6 9 8; 6 2 3 4 5; 9 3 8 9 7; 9 7 4 6 8]
Problem 3 $[C_{ij}]$ 4 ×3= [21 14 7; 15 10 5; 15 10 5; 12 8 4]	**Problem 13** $[C_{ij}]$ 6×10= [10 2 14 9 6 7 21 32 18 11; 7 12 9 3 5 6 9 16 54 12; 4 8 6 12 21 9 21 14 45 13; 21 9 12 9 32 10 19 25 16 10; 10 12 30 15 12 17 30 12 12 9; 15 7 34 17 7 16 14 17 9 5]
Problem 4 $[C_{ij}]$ 4×5= [4 3 6 2 7; 10 12 11 14 16; 4 3 2 1 5; 8 7 6 9 6]	**Problem 14** $[C_{ij}]$ 7×6= [126 207 254 245 214 243; 229 238 242 228 213 285 ; 118 253 306 218 245 216; 172 247 218 248 217 243; 309 207 105 136 194 139; 99 168 220 140 215 116; 95 174 168 145 249 98]
Problem 5 $[C_{ij}]$ 4×6= [44 67 41 53 48 64; 46 69 40 45 45 68; 43 73 37 51 44 62; 50 65 35 50 46 63]	**Problem 15** $[C_{ij}]$ 7×10= [21 11 16 9 15 10 12 32 26 16; 14 15 20 10 16 3 6 9 21 14; 9 17 11 31 21 16 7 9 10 11; 16 23 8 15 10 3 6 3 20 23; 12 40 14 36 9 21 14 19 4 13; 8 18 9 42 8 11 19 9 32 20; 21 9 12 9 32 10 19 25 116 10]
***Problem 6** $[C_{ij}]$ 5×4= [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]	**Problem 16** $[C_{ij}]$ 8×4= [53 62 42 89; 18 35 39 55; 93 80 91 83; 79 23 96 56; 43 16 12 20; 87 70 87 31; 35 79 25 59; 27 16 12 20]
Problem 7 $[C_{ij}]$ 5×4= [9 7 6 2; 6 6 7 6; 5 3 4 4; 4 2 5 9; 2 8 3 6]	**Problem 17** $[C_{ij}]$ 8×5= [300 290 280 290 210; 250 310 290 300 200; 180 190 300 190 180; 320 180 190 240 170; 270 210 190 250 160; 190 200 220 190 140; 220 300 230 180 160; 260 190 260 210 180]
Problem 8 $[C_{ij}]$ 5×6= [10 8 13 20 16 6; 8 16 23 13 14 10; 9 8 1 6 3 7; 4 12 8 11 11 10; 6 10 9 5 11 8]	**Problem 18** $[C_{ij}]$ 10×4= [11 8 9 8; 4 5 29 33; 10 5 29 33; 1 18 25 31; 23 22 33 30; 3 9 13 19; 6 8 27 32; 32 30 39 38; 36 35 31 21; 15 11 10 28]
***Problem 9** $[C_{ij}]$ 5×6= [80 140 80 100 56 98; 48 64 94 126 170 100; 56 80 120 100 70 64; 99 100 100 104 80 90; 64 90 90 60 60 70]	**Problem 19** $[P_{ij}]$ 5×4= [13 15 12 14; 12 14 10 12; 16 18 14 14; 15 15 13 13; 16 15 14 12]
Problem 10 $[C_{ij}]$ 5×8= [300 250 180 320 270 190 220; 290 310 190 180 210 200 300; 280 290 300 190 190 220 230;	**Problem 20** $[C_{ij}]$ 5×6= [12 3 6 -- 5 9; 4 11 -- 5 -- 8 ; 8 2 10 9 7 5; -- 7 8 6 12 10; 5 8 9 4 6 1]

290 300 190 240 250 190 180; 210 200 180 170 160 140 160]	

Note: Problems numbered with 1-18 are UAP of minimization case and 19 are UAP of maximization case and 20 is restricted assignment UAP of minimization case.

Result Analysis:-

To measure the effectiveness of the proposed IDI-75 assignment technique, 20 benchmark instances, listed in Table 10, have been tested and the results are compared with the results of the existing ‘Manta’ technique. The comparison of results is shown in Table 11.

Table 11:- Comparison of results obtained by the ‘IDI-75’ and ‘Mantra’ techniques.

Prob. No. #	IDI-75		Mantra	Prob. No. #	IDI-75		Mantra
	Phase-I	Phase-II			Phase-I	Phase-II	
***1.**	51	50	50	***11.**	16	15	15
2.	21	21	21	***12.**	17	16	16
3.	29	27	27	**13.**	28	28	28
4.	20	20	20	**14.**	881	881	881
5.	168	168	168	**15.**	43	43	43
***6.**	55	54	54	**16.**	73	73	73
7.	10	10	10	**17.**	870	870	870
8.	28	28	28	**18.**	24	24	24
***9.**	328	326	326	**19.**	61	61	61
10.	870	870	870	**20.**	18	18	18

From Table 11, we discover that out of 20 benchmark unbalanced problems tested, the proposed IDI-75 assignment technique has produced OAPs directly to 15 problems through Phase-I itself and for the remaining problems (numbered with 1, 6, 9, 11 and 12), it has produced CAPs which are very close to the OAPs (51, 50), (55, 54), (328, 326), (17, 16) and (16, 15). For these five problems only, we have to go to Phase-II in order to improve the CAP towards OAP. Also, it is noted that the cost matrix for Problem 17 is the transpose of the cost matrix of Problem 10 and for both the problems 10 and 17 the ‘IDI-75’ and ‘Mantra’ techniques have produced the same identical $Z^* =$ \$870. Consequently, the cost matrix of a given UAP (AP) and its transpose produce the OAPs with the same identical Z^*.

Differences between the IDI-75 and Mantra techniques

The important differences between the IDI-75 technique and the ‘Mantra’ technique are listed in the following Table 12.

Table 12:- Differences between the IDI-75 and the ‘Mantra’ assignment techniques.

IDI-75 technique	Mantra technique
This technique works on the principle of reducing the given cost matrix to a matrix of ratio of costs.	This technique works on the principle of reducing the given cost matrix to a matrix of opportunity costs.
It is a ones assignment appoach.	It is a zeros assignment approach.
The elementary arithmetic operation such as ‘division’ only is applied to bring at least one 1-entry in each row or in each column.	The elementary arithmetic operation such as ‘subtraction’ only is applied to bring at least one 0-entry in each row or in each column.
While covering all 1-entries in a reduced ratio of costs matrix by using minimum number of horizontal and vertical lines, it considers the least entry from the uncovered elements and the same is used to ‘divide’ either each entry of the uncovered row only or column only, in which the least entry lies on it.	While covering all 0-entries in a reduced cost matrix by using minimum number of horizontal and vertical lines, it considers the least entry from the uncovered elements and the same is used to ‘subtract’ from either each entry of the uncovered row only or column only, in which the least entry lies on it.
It consists of two phases to solve a given UAP.	It consists of only one phase to solve a given UAP.
If the obtained CAP through the first phase is not optimal, it can be tested for optimality and can be improved	There is no provision for testing the optimality of an obtained CAPs. However, it produces OAPs directly

towards optimality in the second phase.	to all the UAPs.
For the problems numbered with 1, 6, 9, 11 and 12, this technique has produced near OAPs only through the first phase and one has to apply the second phase to get the OAPs.	This technique has produced OAPs directly to all the UAPs.

Conclusion:-

In this article, we have presented a simplified form of the existing Ones Assignment Method named IDI-75 to find an optimal pattern of assignments to UAPs. The presented technique IDI-75 has been implemented on 20 benchmark UAPs (minimization, maximization and restricted assignment cases) in different sizes from the literatures. Simulation results substantiate that IDI-75 is the best technique which produces optimal solution to all the 20 instances. Hence, in case of 'ones assignment approach', it is intelligent to apply the IDI-75 technique to solve the UAPs. Also, this technique will be more cost-effective for those decision makers who are dealing with assignment problems which are of unbalanced category.

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Appendix-A

'ME Rules' for covering all 1s by Lines

The word ME is coined from the first letter of the names of the designed authors Murugesan and Esakkiammal. In this section, we present a new set of rules, called 'ME rules', of how to draw the minimum number of horizontal and vertical lines to cover all the 1s of a 'ratio of costs matrix' (RCM) and also to achieve a 'complete assignment plan' (CAP).

Rule 1: To draw Minimum number of Lines to cover all 1s

a) Row-wise assignment

(i) Look at the rows successively from first to last until a row with exactly one 1 entry is found.

(ii) Make an assignment to this single 1 entry by creating a circle or square around it.

(iii) Draw a vertical line passing through that 1 entry.

(iv) Continue in this way until all the rows have been scrutinized.

(v) After scrutinizing the last row, check whether all the 1s are covered with the drawn lines. If yes, go to Rule (2); otherwise, do column-wise assignment.

b) Column-wise assignment

(i) Look at the columns successively from first to last until a column with exactly one unassigned 1 entry is found.

(ii) Make an assignment to this single 1 entry by creating a circle or a square around it.
(iii) Draw a horizontal line passing through that 1 entry.
(iv) Continue in this way until all the columns have been scrutinized.
(v) After scrutinizing the last column, check whether all the 1s are covered with the overall drawn lines. If yes, go to Rule (2); otherwise, do row-wise and column-wise assignments, if possible. Then go to Rule (2)

/* By a ‘complete assignment plan’ (CAP) for a cost matrix of order $m \times n$, we mean an assignment plan or program containing exactly k assigned independent 1s, where $k = \text{Min}\{m, n\}$, one in each row when $m < n$ and one in each column when $n < m$. A CAP for an UAP is said to be ‘achieved’ if the following two conditions are satisfied:
(a) Minimum number of lines drawn must be exactly equal to k, where $k = \text{Min}\{m, n\}$.
(b) Each row (column) of the assignment matrix must have a unique ones assignment when $m < n$ ($n < m$) */

Rule 2: To test the conditions for CAP

Test whether the conditions for CAP is achieved. If yes, write the CAP and compute the corresponding ‘overall assignment cost (or profit)’; otherwise, select the smallest (largest) element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered row(s) or column(s), which d_{ij} lies on it. This operation creates some new ones to this row or column and hence a revised RCM is obtained. Then, go to Rule 1.

If the conditions for CAP are not satisfied through the above said two rues, then apply rule 3.

Rule 3:

After performing the row-wise assignment and column-wise assignment completely as far as possible in the revised RCM, if more than one 1s are present in certain rows and columns, then
(i) Select any one 1 entry arbitrarily and make an assignment to that 1 entry by creating a circle or square around it.
(ii) Draw a horizontal line through the row of the assigned 1 entry and put an X mark on all the remaining 1s on the column of that assigned 1 entry. (Or) Draw a vertical line through the column of the assigned 1 entry and put an X mark on all the remaining 1s on the row of that assigned 1 entry.
(iii) Repeat (i) and (ii) until the conditions for a complete assignment are satisfied.
The situation of applying Rule 3 creates an alternative CAP to the given UAP.