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*RESEARCH ARTICLE*

# DC RAILWAY POWER FLOW ANALYSIS FOR ADDIS ABABA LIGHT RAIL TRANSIT USING NEWTON RAPHSON METHOD

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***Abstract***

This paper uses Newton–Raphson method for DC power flow analysis of the Addis Ababa light Rail Transit (AALRT). The study focuses onthe line section from Menilik II square station up to Lideta station. First the tractive effort required by the trains for different scenarios such as train movement in a straight line, a curved line, and a line with gradient is computed as the chosen line section contains all these scenarios. Then the total input power will be calculated using computed tractive effort obtained for each scenario and using other input parameters obtained from AALRT, and different papers. The input power for the different loads is computed, and the input power is used to analyse the bus voltage for different loads and train positions. Newton Raphson Method is used to solve the DC Power bus problem assuming that the train requires constant power while moving between two feeding stations. Even if using the rail as the return conductor for DC traction systems has economic advantages, it has some limitations like the rail potential and stray current. A rail potential study is carried out and conclusions are drawn. The result shows that the maximum voltage drop was 0.1 p.u and the train power consumption increases by 136.73 kW as the train takes a gradient of 3.92% and keep increasing again by 29.17kw with a curve resistance (100 meters). The Rail potential moves from 6.0139V to 29.85V proportionally with the variation of the total ground resistance.



## Introduction:-

In recent decades, demand growth in public transport systems has increased rapidly. Several cities across the world have planned to develop their own urban mass transit systems or to extend their existing routes to cover every street corner. Most urban metro systems require DC traction power supply to energize their rail vehicles. The third rail conductor in DC power feeding systems is typically used for urban metros with the standard DC supply voltage of 750 V [1, 2 ,3]. At higher voltage level, 1500 VDC or 3000 VDC, the overhead catenary feeding configuration is more appropriate. It is necessary to characterize electrical performance and power loading at traction substations for the planning, designing, and operation of mass rapid transit. DC railway power flow calculation has been continually developed. Some may consider that DC railway power flow is a reduced version of AC power flow. As AC power flow, Gauss-Seidel and Newton–Raphson methods are both well-known and widely accepted. In DC railway power systems, these two methods have been commonly employed in the case of non-linear traction power load [4, 5]. The nature of the DC railway power system is as simple as DC linear circuits unless a traction power load model is

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considered. A study of peak power demand and energy consumed by each traction substation needs to be determined to verify that the electrical energy flowing in its railway power feeding system is appropriate or not. Gauss-Seidel, conventional Newton–Raphson, and current injection methods are well-known and widely accepted as a tool for electrical power network solver in DC railway power supply study [6, 7]. In this paper, a Newton–Raphson method has been proposed.

In a railway traction power supply system, the transmission lines should not be overloaded, the catenary voltage should be within specified voltage level, the train power requirements should meet, and the system should operate effectively and efficiently. In an electrified railway traction power supply system, the load in the electrical circuit is the locomotives that are moving and demanding different levels of power according to their dynamic characteristics, operational mode, and speeds. So that, due to load variations, voltage drops in the feeding circuit differ substantially depending upon the train position, train current, number of trains in the same power feeding section, track impendence, etc. On the other hand, power consumption problems have occurred frequently within a large variety of loads. These issues are important in the traction power supply system to ensure the normal operation of the electric locomotives. Thus, to address the above requirements this paper presents DC railways power flow analysis using the Newton Raphson method on AALRT.

This can be achieved through the following specific objectives:
Determine the DC bus voltages for different train positions.
Determine the effect of changing the load on the DC bus voltage output.
Determine the effect of ground resistance on the DC bus voltage.

## Literature Review & Theoretical Background:-

Over years, numerous studies and simulation of dc power flow analysis has been done. Kulworawanichpong [8] not only solved the power flow of a multi-train model but also avoided the complication that arose due to using derivatives within matrices by adopting a simplified Newton-Raphson method. In the conventional methods, the Jacobian matrix needs to be recalculated iteratively [8]. However, when the simplified Newton-Raphson method is adopted, the off-diagonal elements are only calculated once and their values remain fixed while the diagonal elements are recalculated at every iteration.

Mohamed, and Al in [9] proposed a method for solving the electric railway power flow in DC traction, which they denoted as a Modified Current Injection (MCI) method. This method is a modification of existing algorithms aiming primarily to improve the convergence and reduce the solving time. MCI can also be applied to solve the electric railway power flow whether the substation is reversible or non-reversible. The drawback of this method starts from the need to provide the initial voltage profiles for the nodes in order to calculate the currents injected into the nodes.

Jabr and Al in [10] solved the power flow of a DC railway with 144 running trains by using current injection method, commonly known as the Conductance Matrix iterative (CMI) method. This approach is beneficial, as it avoids overvoltage by adjusting the regenerative energy that is reinjected through to bi-directional substations. Moreover, when the substations are no bidirectional (rectification only), they are switched off and the regenerating trains are switched to voltage-current sources. Its main objective being to reduce the number of required iterations, thus increasing the speed which the system solution is generated.

Different simulation methods for electric railways have been proposes in extant literature [11, 12, 13, 14]. However, these approaches provide only the voltages experienced by the trains and not the track voltage. Authors of [15] suggested a simulation method that uses MATLAB Simulink to solve the power flow of an electric railway with seven substations. The proposed model measured the traction substations voltages and currents. Simulation of a single train running on the Sukhumvit Line by Mongkoldee, Kulworawanichpong and Leeton to calculate the voltage at the train's location, traction substations power and power losses in the transmission line. The system was modelled by a MATLAB M-file and the power flow was solved using the current injection method.

Arboleya, and Al in [16] avoided the variation of the railway power network by using graph theory. Later on, simple matrix formulation was developed to solve the power flow. The proposed approach exhibited high accuracy compared to that of the commercial software DIgSILENT Several railway simulations tools have been provided by the industry such as Vitas, Trainops, SitrasSidytrac and TOM.

**DC Railway Power Supply system**

The commonly used voltages for DC railways are 600 V, 750 V, and 1.5 kV for urban, interurban metros, and regional system, respectively. Overhead catenary is typically used for light rail system at 600–800 V and for conventional interurban or regional systems at 1.5 or 3 kV. Because of the large currents involved compared to high voltage AC system, the DC copper contact wire is made from heavier gage material. DC railway power supply system has several configuration features different from a normal DC power system. However, there are some simplifications of the power network modelling. DC feeding system feature Figure 1, includes a three-phase bridged silicon rectifier for conversion from alternating to direct current. And Figure 2 represents an equivalent circuit of a DC railway traction power system.

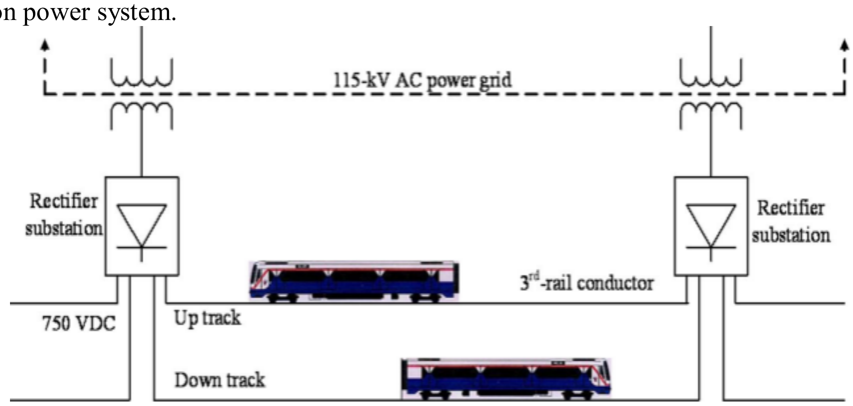



**Figure 1:-** Circuit diagram of a typical DC railway power system.

The DC railway power supply system is not a simple DC linear circuit due to two causes of non-linearity. The first is the rectifier substation that does not allow the current flowing in the negative direction. The second is the traction power of the train.

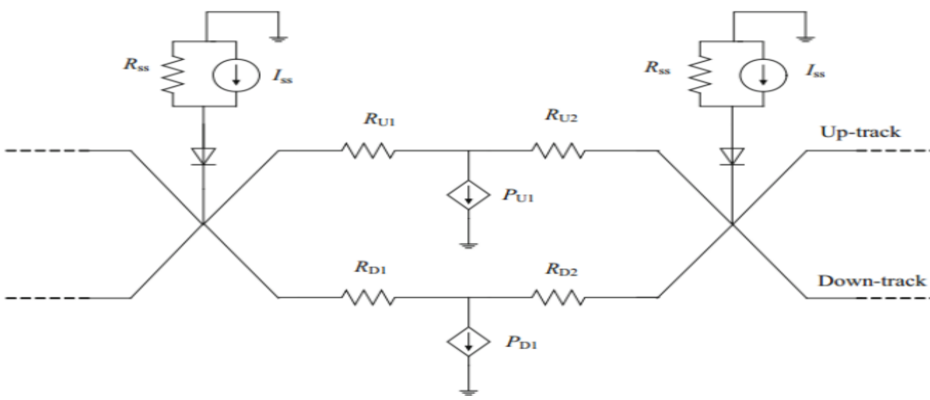



**Figure 2:-** Equivalent circuit of a DC railway traction power system.

The rectifier substation is modelled by Norton's equivalent source in which Iss and Rss represent the Norton's short-circuit current and the Norton's resistance, respectively; RU1 and RU2 are the conductor resistances of the up-track sections; RD1 and RD2 are the conductor resistances of the down-track sections; PU1 and PD1 are the power consumptions of the running trains on the up-track and the down-track. The diode placed at the substation terminal is used to prevent any negative current flowing into the substation. The train model is represented by a controlled current model, IT = PT/VT. Hence, the DC power flow equation at bus k can be described as follows:

$$V_k I_{ss,k} - P_{T,k} = V_k \sum_{i=1}^{n} G_{k,i} V_i \quad (1)$$

Where $V_k$ is the voltage at bus k, $I_{ss,k}$ is the short-circuit capacity of the substation at bus k, Pk is the power load of the train connected at bus k, $G_{k,i}$ is an element k,i of the bus conductance matrix.

**Conventional Newton-Raphson DC railway power flow solution**

As the same as for AC power flow calculation [17], the updated voltage is calculated using Taylor series expansion of the power mismatches. With first order derivates of equation 1, the Jacobian matrix,
$[J] = [\partial \Delta P / \partial V]$ , can be formulated.

$\Delta P\,k \ = Vk - \ PT,k \ - Vk\sum_{i=1}^{N} Gk,i\,Vi$ (2)
$(\partial/\,\partial Vk)\Delta P\,k \ = Iss,k - 2VkGk,k - \sum_{i=1,i\neq k}^{N} Gk,i\,Vi$ (3)
$(\partial/\,\partial Vk)\Delta Pk = -VkGk,j$ (4)

For DC railway power system, equation 3 and 4 for (for diagonal and off-diagonal elements, respectively) are used to compute elements of the Jacobian matrix. Therefore, the updates voltage at bus k of the h+1 iteration can be found in equation 5.
$[V]^{(h+1)} = [V]^{(h)} - [(\partial/\,\partial Vk)\Delta P]^{-1}[\Delta P]$ (5)

**Power system of The Addis Ababa Light rail Transit**
First Let us specify and calculate the input parameter that can be used for DC power bus analysis of Addis Ababa Light Rail Transit. The rectifier, the transformer configuration used in Addis Ababa light Rail Transit has the following configuration Figure 3.

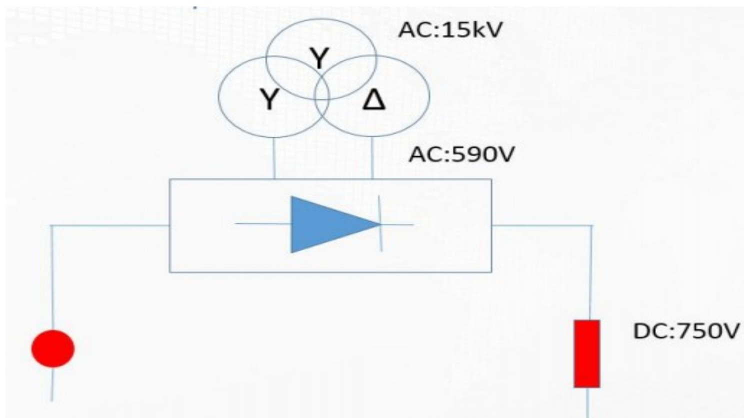



**Figure 3:-** Transformer-Rectifier arrangement for AALRT.

Under normal circumstances, light rail, at an interval of 1 to 2 stations, set up a traction step-down substation, traction step-down hybrid substation has function of both traction and step-down, on the one hand, the main transformer substation 15kV AC supply, by means of rectifier transformer and rectifier, is converted to 750V DC, energizing the catenary power supply. First the 15 KV AC supply from the power grid is stepped down to 590 V AC. The primary of the transformer is in star connection while the output AC voltage, the secondary is in both star and delta configurations. The output AC voltage is then fed to a rectifier, and the rectifier gives a 750 DC voltage. The detailed interconnection of the output AC voltage with the 12-pulse rectifier is shown in the figure 4below.

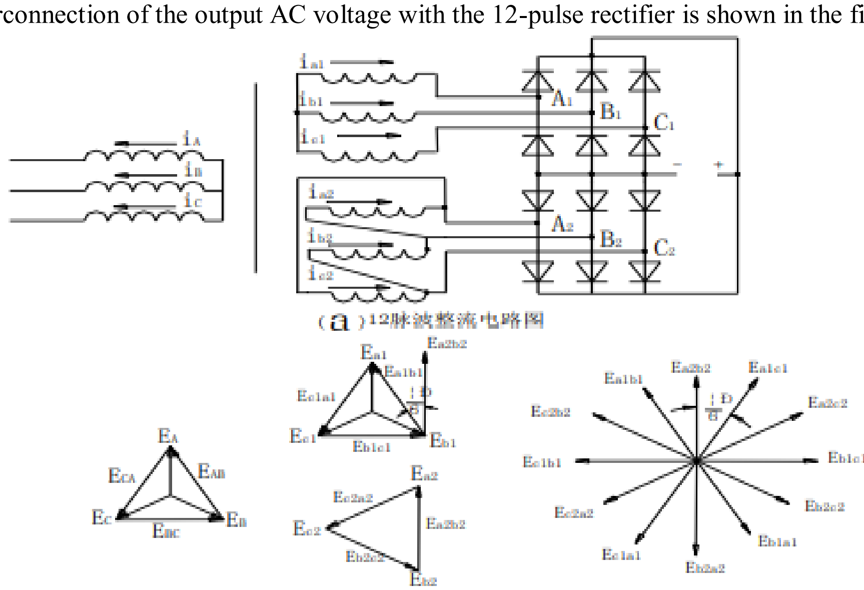



**Figure 4:-** Detailed Configuration for Transformer-Rectifier arrangement for AALRT.

The wiring diagram for both North- South and East-West lines is given below. This diagram was obtained directly from Addis Ababa Light Rail Transit. As it can be seen from Figure 5 configuration, each phase from the AC substations is given the three subsequent stations that are close to it and the others are interconnected using a loop-in and loop out connection.

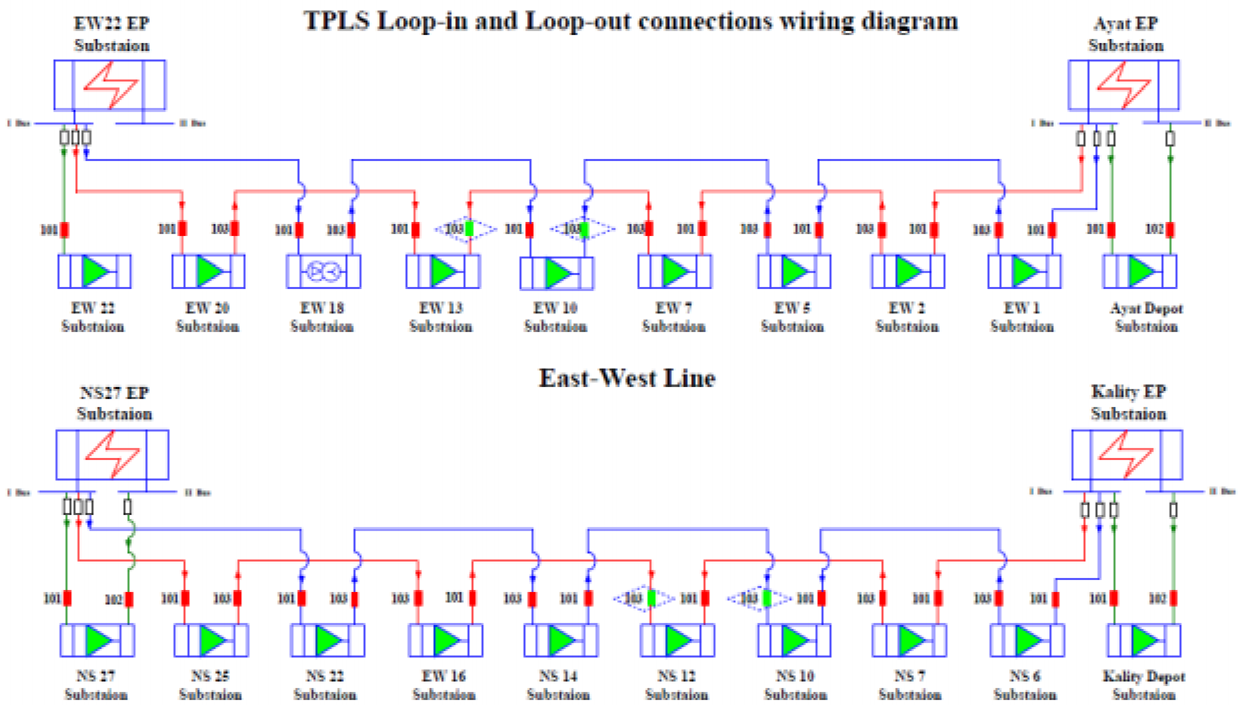



**Figure 5:-** Wiring diagram of the AALRT feeding station.

The location of each feeding substation along the line is shown below

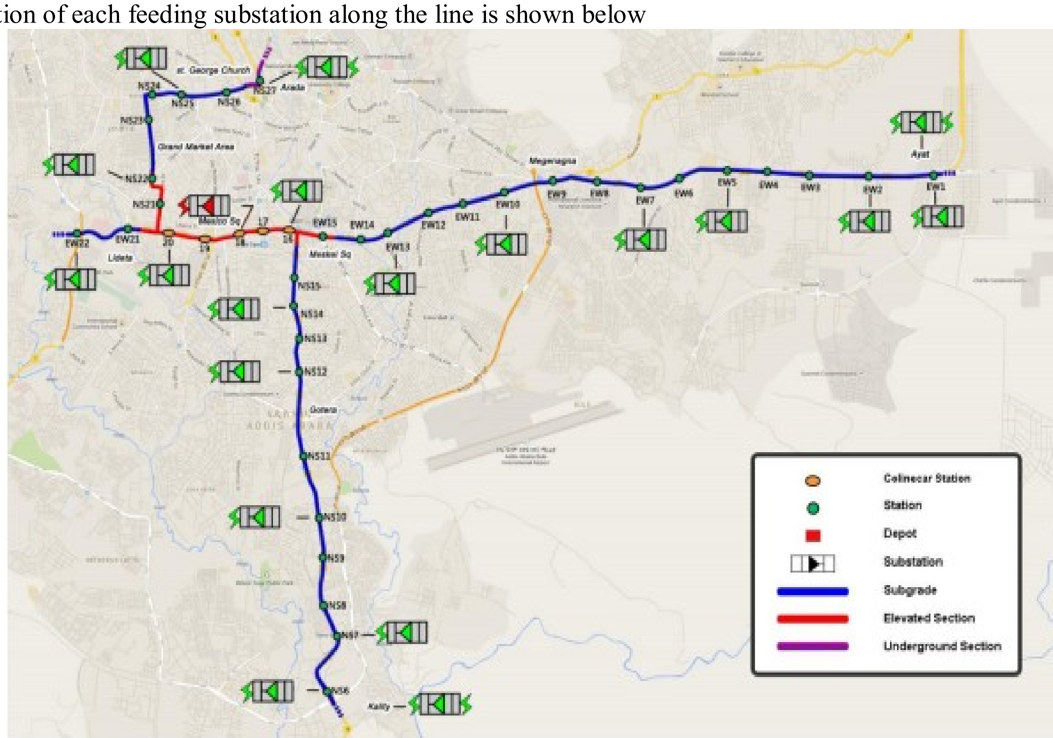



**Figure 6:-** Location of the feeding stations along the line.

For this Analysis, the four feeding stations, NS27, NS25, NS 22 and EW16, starting from the Menilik II square AC substation, NS27EP are considered. This section is chosen to see both the effects of horizontal curvatures and vertical gradient on the power consumption of the train.

The selected feeding station feed power for the line starting from Menilik II square up to St. Lideta as shown in the blue line in the figure below. Even if a single feeding station may supply power for 1 or 2 stations, for simplicity only one station is considered per feeding station in this analysis. As a result, the passenger stations Atikilt Tera, Sebategna, Autobus Tera and Darmar are not considered. The feeding station are present only on Menilik II, GojamBerenda, Abnet and St. Lideta stations. So, for ease of analysis only these stations are considered as shown in the figure 6 by the blue, green and yellow line.

The first line section that is between Menilik II square and GojamBerenda is the one shown in the blue line in the figure above and it is 1686 m long. The second line section that is between GojamBerenda and Abnet is the one shown in the green line in the figure above and it is 2089 m long. The third line section that is between Abnet and lideta is the one shown in the yellow line in the figure above and it is 1330 m long. The first line section is assumed to be straight, and the second line section is assumed to be with a gradient of 2 in 100. And the third line section is assumed to be with a curve of radius of 50 meter. These values are the ones that are used in the project.

**Design and Calculation**

The following input parameters were obtained from MSc Thesis done by Eshetu Shewalema [3] and Abebe Teklu [1] on Addis Ababa Light Rail Transit. In addition, some of the data was collected from Addis Ababa Light Rail Transit directly

**Table 1:-** Different input parameters for the AALRT.

| No. | Parameter | Value |
|---|---|---|
| 1 | Train car weigh | 44tone |
| 2 | Vehicle Car weight with full passenger 6persons/m2 | 59.24tone |
| 3 | (With 8 persons/m2) | (63.02tone) |
| 4 | Vehicle length (single vehicle) | 29.4m |
| 5 | Car body width | 2.65m |
| 6 | Maximum operation speed | 70km/hr |
| 7 | Operation base speed | 40km/hr |
| 8 | Acceleration under rated load (from 0-40km/hr) | 1m/s2 |
| 9 | Acceleration under rated load (from 0-70km/hr) | 0.5m/s2 |
| 10 | Motor efficiency ($\eta_{motor}$) | 0.87 |
| 11 | Inverter efficiency ($\eta_{inv}$) | 0.96 |
| 12 | Gear box efficiency ($\eta_{gear}$) | 0.90 |

**Train Demand Supply Analysis**

In order to compute the energy requirements for a train operating on a defined track, the standard equations of motion are applied. To apply these equations, the forces acting against the train movement have to be taken into account. These forces include rolling resistance, resistance due to aerodynamic drag, resistance due to acceleration and resistance due to gradient. In our configuration, we have a pair of substations connected by a line with gradient and one pair of substations connected by a curved line as shown if figure 7. Therefore, for simplicity, in this model, we will consider four opposing forces namely: acceleration force, train motion resistance, resistance due to the gradient and resistance due to curve.

This Time is important because it help us determine the average power for each case as the total energy output is done during this time. The rated passenger capacity is considered for most of the cases to be evaluated. In this case, the total number of passengers is 254, and the average weight of the passengers is 60 Kg. So, the total rated passenger capacity 60 x 254 = 15240. And the total weight of the train for this case is the sum of the empty train with and the rated passenger capacity. Therefore, the total train weight is given by: Total Weight = Empty Train Weight + Rated Passenger Capacity. M = 15240 + 44000 = 59240 Kg = 59.24 Tone

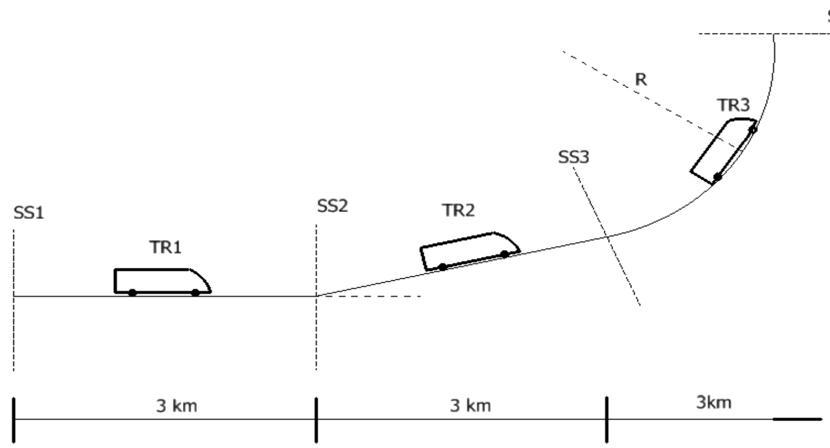



**Figure 7:-** Line configuration with a gradient and a curve.

The tractive effort required for train propulsion is expressed in [18] as:

$$F_t = F_a + F_g + F_r + F_c \qquad (6)$$

where $F_a$ is the force required for giving linear acceleration to the train.

3.1.1 Force required to overcome resistance due to acceleration (Fa)

As stated by the Newton second law of motion, the force required to accelerate the train in motion is given by:

$$\text{Force} = \text{Mass} \times \text{acceleration} \qquad (7)$$

The fact that the train has rotating parts such as motor armature, wheels, axels, and gear system, its mass including the mass of rotating parts is known as effective mass or accelerating mass (Me) and is much higher (about 8–15%) than its stationary mass (M). Hence, these parts need to be given angular acceleration at the same time as the whole train is accelerated in a linear direction. Thus, the equation above becomes:

$$F_a = M_e \times \alpha \qquad (8)$$

When the train effective mass $M_e$ is expressed in kg and acceleration α expressed in m/s2 the above equation becomes:

$$F_a = 1000M_e\left(\frac{1000}{3600}\right)\alpha\left(\text{kg} - \text{m}/_{\text{s}^2}\right) \qquad (9)$$

Therefore, the force required for a linear and angular acceleration is [16]:

$$F_a = 277.8M_e \times \alpha \quad (N) \qquad (10)$$

**Force required to overcome resistance (Fr)**

When the train is running at uniform speed on a level track, it has to surmount the opposing forces due to the surface friction, i.e., the friction at various parts of the rolling stock, the fraction at the track, and also due to the wind resistance. Parameters that affect or determine the size of the frictional resistance include shape, size, track conditions, train speed, etc. Considering r as specific resistance, the force required to overcome this resistance is:

$$F_r = Mr,$$
$$r = \frac{R}{M} \qquad (11)$$
$$F_r = R.$$

The total resistance against the train movement is given by Davis equation:

$$R = 1.3W + 29N + b \times W \times V + c \times A \times V^2 \qquad (12)$$

Where R is the total resistance in lbs. (1lbs = 4.45N); W is the train weight in tones; N is the number of train axles; V is the train speed in miles/hour; b is the experimental friction coefficient (for passenger car b = 0.03); c is drag coefficient (for passenger car); A is the cross section of train frontal area (square feet); At maximum speed (70km/h equivalent to 112 mph)

**Force required to overcome the gradient resistance (Fg)**

When the train is moving on up gradient, Figure 8, the gravity component of the dead mass opposes the motion of the train in an upward direction. In order to prevent this opposition, the force should be acting in an upward direction [16].

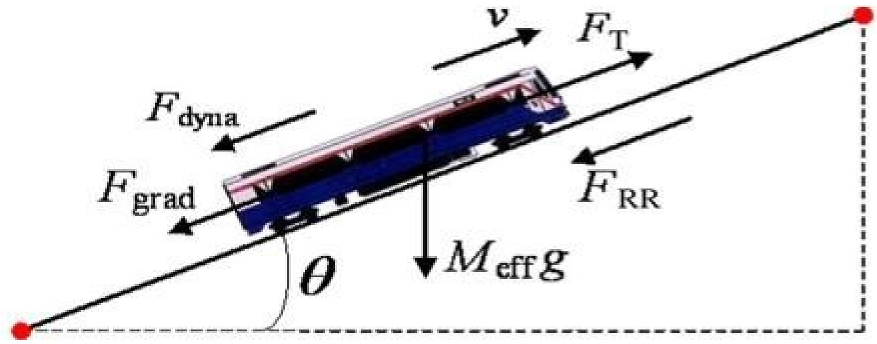



**Figure 8:-** Resistance due to gradient.

Considering an uphill motion of the train, the force required to overcome the resistance due to gravity is:

$$F_g = M_g \sin\theta \tag{13}$$

In railway practice, the gradient is expressed as the rise (in meters) a track distance of 100m and is called "percentage gradient".

$$\%G = \frac{BC}{AC/100} = 100 \times \frac{BC}{AC} = 100 \sin\theta \tag{14}$$

Substituting the value of sin θ, the equation becomes:

$$F_g = M_g \frac{C}{100} \tag{15}$$

When the mass is expressed in kg, the force to overcome gradient becomes:

$$F_g = 9.81 \times 10^{-2}(1000MG)$$
$$= 98.1MG \ \text{(Newton)}$$

3.1.4 Force required to overcome the curve resistance (Fc)

This resistance is related to the wheel flange friction as the result of curving radius R in meters as shown in Figure 9.

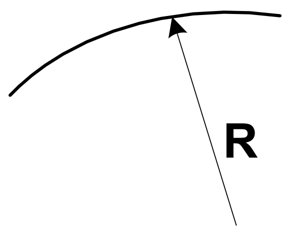



**Figure 9:-** Resistance due to curve-radium.

The empirical formula is given by:

$F_C = 9.81(700/R)M$ Newtons

Where M is the weight of the train in tones.

Another approximation empirical formula for curve radii above 150 m is given by:

$F_C = m\,[(6.5m^2/s^2/(R-55)]$ Newtons

Where m- train mass in kg and R- radius of curve in meters

3.2 Power calculation

The power drawn from the feeding substations system should be equal to the power consumed by the various parts of the train and the quantity of the energy required for lighting, heating, control, and braking. The train power consumption calculation requires detailed train running states, train speed, running time and corresponding tractive effort. In this model, the power is calculated as a function of the force and speed as:

$P = F_t \times V$

Where $F_t$ is the tractive effort required for train propulsion and V is the train speed.

For the three scenarios considered in figure 8, we have three different tractive efforts where:

$$F_1 = F_a + F_r$$
$$F_1 = F_a + F_r + F_g$$

$$F_1 = F_a + F_r + F_g + F_c$$

The corresponding Powers are calculated as:

$$P_1 = F_1 \times V$$

$$P_2 = F_2 \times V(16)$$

$$P_3 = F_3 \times V$$

**Problem solving using Newton Raphson method**

The objective of a power flow study is to calculate the voltages (magnitude and angle) for a given load, generation, and network condition. Once voltages are known for all buses, line flows and losses can be calculated. The starting point of solving power flow problems is to identify the known and unknown variables in the system. The approach to solving the power flow problem is to use an iterative algorithm. The Newton-Raphson algorithm is the most commonly used algorithm in commercial power flow programs. Starting with a reasonable guess at the solution, this algorithm checks to see how close the solution is, and then if it is not close enough, updates the solution in a direction that is sure to improve it, and then repeats the check. This process continues until the check is satisfied. Usually, this process requires 5-20 iterations to converge to a satisfactory solution. For large networks, it is computationally intensive. Figure 10 shows the circuit diagram for the power flow analysis of our system (Train's direction is from substation 1 to substation 4).

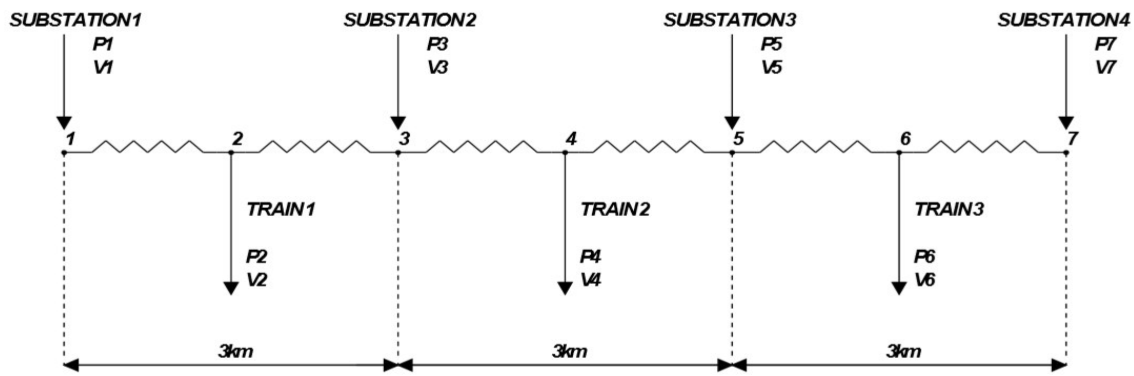



**Figure 10:-** Network configuration for the power flow analysis.

The conductance matrix of a power system is an abstract mathematical model of the system. It consists of conductance values of both lines and buses. The Y-bus is a square matrix with dimensions equal to the number of buses. For our particular model, the admittance matrix a $(7 \times 7)$ square matrix since our system consists of 7 nodes as seen in figure10. This matrix is symmetrical along the diagonal.

**Table 2:-** Nodes Characteristics.

| Node | Known (in p.u) | Unknown |
|---|---|---|
| 1 | V1=1 | P1=? |
| 2 | P2= -0.4973 | V2=? |
| 3 | V3= 1 | P3=? |
| 4 | P4= -0.6610 | V4=? |
| 5 | V5=1 | P5=? |
| 6 | P6= -0.6902 | V6=? |
| 7 | V7=1 | P7=? |

Note that the values are in per unity system with base voltage (VB)= 750V and base power (PB)=1000kW

With the seven nodes in the system, a 7 by 7 conductance matrix is formulated, given by:

$$Y_C = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} & Y_{17} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} & Y_{27} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} & Y_{36} & Y_{37} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} & Y_{46} & Y_{47} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} & Y_{57} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} & Y_{65} & Y_{66} & Y_{67} \\ Y_{71} & Y_{72} & Y_{73} & Y_{74} & Y_{75} & Y_{76} & Y_{77} \end{pmatrix}$$

the power at any bus is expressed mathematically as:

$P_i = V_i \sum_{j=1}^{7} V_i Y_{ij} (17)$

Power at bus 2 is:

$$P_2 = V_2(V_1Y_{21} + V_2Y_{22} + V_3Y_{23} + V_4Y_{24} + V_5Y_{25} + V_6Y_{26} + V_7Y_{27})$$

Power at bus 4 is:

$$P_4 = V_4(V_1Y_{41} + V_2Y_{42} + V_3Y_{43} + V_4Y_{44} + V_5Y_{45} + V_6Y_{46} + V_7Y_{47})$$

Power at bus 6 is:

$$P_6 = V_6(V_1Y_{61} + V_2Y_{62} + V_3Y_{63} + V_4Y_{64} + V_5Y_{65} + V_6Y_{66} + V_7Y_{67})$$

To get the Jacobian matrix, the bus powers are differentiated with all unknowns, that is, $V_2$,$V_4$ and $V_6$.

$\frac{\partial P_2}{\partial V_2} = V_1Y_{21} + 2V_2Y_{22} + V_3Y_{23}; \frac{\partial P_2}{\partial V_4} = 0\ ; \frac{\partial P_2}{\partial V_6} = 0;$

$\frac{\partial P_4}{\partial V_2} = 0; \frac{\partial P_4}{\partial V_4} = V_3Y_{43} + 2V_4Y_{44} + V_5Y_{45}\ ; \frac{\partial P_4}{\partial V_6} = 0;$

$\frac{\partial P_6}{\partial V_2} = 0; \frac{\partial P_6}{\partial V_4} = 0; \frac{\partial P_6}{\partial V_6} = V_5Y_{65} + 2V_6Y_{66} + V_7Y_{67};$

Hence the Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_4} & \frac{\partial P_2}{\partial V_6} \\ \frac{\partial P_4}{\partial V_2} & \frac{\partial P_4}{\partial V_4} & \frac{\partial P_4}{\partial V_6} \\ \frac{\partial P_6}{\partial V_2} & \frac{\partial P_6}{\partial V_4} & \frac{\partial P_6}{\partial V_6} \end{bmatrix}$$

To solve the circuit using iterative method, we use: $V^{n+1} = V^n - (J^{-1} \times \Delta P)$ which after 10 iterations gives convergence of the respective voltages of the buses.

Where $\Delta P = J\Delta V$ and making $\Delta V$ the subject we have; $\Delta V = J^{-1}\Delta P$.

For the starting point, we assume $V^{(0)} = \begin{matrix} V_2^{(0)} \\ V_4^{(0)} \\ V_6^{(0)} \end{matrix} = \begin{matrix} 1 \\ 1 \\ 1 \end{matrix}$ , which the values are in per unit

Depending on the train location from each of the two nearest substations, Voltages vary accordingly. In order to consider the effect of train position, we have considered 3 cases referred to as case 1, case 2 and case 3. For each of the three cases, the conductance matrix and bus voltages are calculated as follow:

Case 1: considering when the trains are at the midpoint of the substation, that is 1.5 km, and taking the $R_{rail} = 0.02\Omega/km$, we compute the conductance matrix to be:

$$Y_C = \begin{pmatrix} 33.333 & -33.333 & 0 & 0 & 0 & 0 & 0 \\ -33.333 & 66.667 & -33.333 & 0 & 0 & 0 & 0 \\ 0 & -33.333 & 66.667 & -33.333 & 0 & 0 & 0 \\ 0 & 0 & -33.333 & 66.667 & -33.333 & 0 & 0 \\ 0 & 0 & 0 & -33.333 & 66.667 & -33.333 & 0 \\ 0 & 0 & 0 & 0 & -33.333 & 66.667 & -33.333 \\ 0 & 0 & 0 & 0 & 0 & -33.333 & 33.333 \end{pmatrix}$$

Case 2: considering when train 1 is at 1.8 km ahead of substation 1 and 1.2 behind substation 2; Train 2 being located at 1.3 km ahead of substation 2 and 1.7 km behind substation 3; train 3 located at 2.5 km ahead of substation 3 and 0.5 km behind substation 4. taking the $R_{rail} = 0.02\Omega/km$ we compute the conductance matrix to be:

$$Y_C = \begin{pmatrix} 27.778 & -27.778 & 0 & 0 & 0 & 0 & 0 \\ -27.778 & 69.445 & -41.667 & 0 & 0 & 0 & 0 \\ 0 & -41.667 & 80.129 & -38.462 & 0 & 0 & 0 \\ 0 & 0 & --38.462 & 67.874 & -29.412 & 0 & 0 \\ 0 & 0 & 0 & -29.412 & 49.412 & -20 & 0 \\ 0 & 0 & 0 & 0 & --20 & 120 & -100 \\ 0 & 0 & 0 & 0 & 0 & -100 & 100 \end{pmatrix}$$

Case 3: considering when the trains are at 2 km ahead of substations and 1 km behind substations, that is 2/3 of the section. taking the $R_{rail} = 0.02\Omega/\text{km}$ we compute the conductance matrix to be:

$$Y_C = \begin{pmatrix} 25 & -25 & 0 & 0 & 0 & 0 & 0 \\ -25 & 75 & -50 & 0 & 0 & 0 & 0 \\ 0 & -50 & 75 & -25 & 0 & 0 & 0 \\ 0 & 0 & -25 & 75 & -50 & 0 & 0 \\ 0 & 0 & 0 & -50 & 75 & -25 & 0 \\ 0 & 0 & 0 & 0 & -25 & 75 & -50 \\ 0 & 0 & 0 & 0 & 0 & -50 & 50 \end{pmatrix}$$

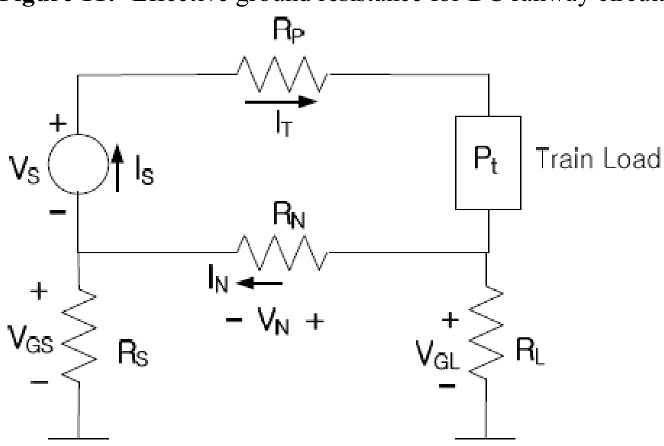
Using the iteration formulas stated earlier and running the code in MATLAB (10 iterations), the bus voltage computed can be obtained.

**Effect of Rail Potential**

In most rail transit systems, the running rails are used as the return conductor for traction current. Traction current from drawn from the catenary through the pantograph returns to traction substation through return conductor rails. This arrangement has the distinctively economic advantage as no dedicated return conductor is required. The disadvantages associatedwithsuchanarrangementarerailpotentialandstraycurrentproblems.Therailpotentialis the voltage occurring under operating conditions when the running rails are utilized for carrying the traction return current or under fault conditions between running rails andearth.

The maximal rail potential always exists at locations of loads or traction substations, and almost decreasing to zero outside the transition area. Rail potential is influenced by some factors, such as the conductance per unit length betweenrunningrailsandearth,weather,tractioncurrent,distancebetweentractionloadandtractionsubstation.The conductance between running rails and earth has great influence on railpotential [19].

**Figure 11:-** Effective ground resistance for DC railway circuit




Using Kirchhoff's voltage rule applied to Figure 11, we have:

$$V_{GS} + V_N - V_{GL} = 0$$
$$V_{GS} = V_{GL} - V_N$$

Since the grounds are connected to the same point, due to conservation of charges,

$$I_{GS} + I_{GL} = 0$$
$$I_{GS} = -I_{GL}$$

But $I_{GS} = \frac{V_{GS}}{R_S}$ and $I_{GL} = \frac{V_{GL}}{R_L}$

Therefore, the above relationships, we obtain,

$\frac{V_{GS}}{R_S} = -\frac{V_{GL}}{R_L}$

Rearranging the terms, we obtain

$$V_{GL} = -R_L \frac{V_{GS}}{R_S}$$

$$V_N = -R_L \frac{V_{GS}}{R_S} - V_{GS}$$

$$V_N = -V_{GS} \left(\frac{R_L}{R_S} + 1\right)$$

$$V_{GS} = -V_N \frac{1}{\left(\frac{R_L}{R_S} + 1\right)}$$

$$V_{GS} = -V_N \frac{R_S}{(R_S + R_L)}$$

$$V_N = I_N R_N$$

$V_{GS} = -I_N R_N \frac{R_S}{(R_S+R_L)}$(18)

Thus

$$V_{GL} = I_N R_N \frac{R_L}{(R_S + R_L)} \quad (19)$$

## Results and Discussion:-

The results on the power flow analysis are obtained by running the Matlab code considering 10 iterations. The output results are listed in the table 3 and 4 which show respectively the trains power consumption at different tract location and Voltage drop.

**Table 3:-** Trains power consumption.

| Section | Scenario | Power consumption | |
|---|---|---|---|
| 1-2 | Train is between Substation 1 and 2. The power consumption by the train is due to acceleration resistance and train friction resistance only | P2(kW) | 497.3 |
| 2-3 | Train is between substation 2 and 3. The power consumption by the train is due to acceleration resistance, train friction resistance and gradient resistance (G=3.92%) | P4(kW) | 661.03 |
| 3-4 | Train is between substation 2 and 3. The power consumption by the train is due to acceleration resistance, train friction resistance, a gradient resistance (G=3.92%) and curve resistance (100 meter). | P6(kW) | 690.2 |

**Table 4:-** DC Voltage for different train positions.

| Distance Travailed as percentage of total distance | Computed Bus Voltage (in p.u) | | |
|---|---|---|---|
| | V2 | V4 | V6 |
| First scenario | 0.9924 | 0.99 | 0.9895 |
| Second scenario | 0.9933 | 0.9911 | 0.9907 |
| Third scenario | 0.9928 | 0.9902 | 0.9942 |

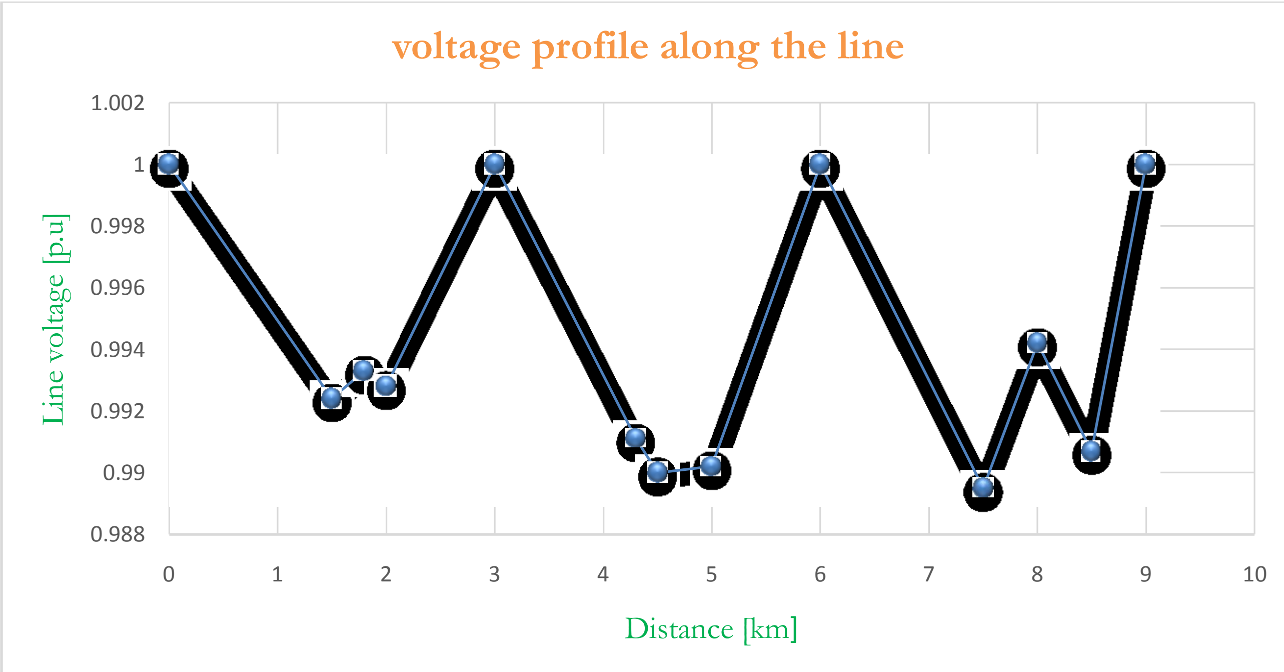



**Figure 12:-**Voltage profile along the catenary.

From figure above we can easily see the variations of the catenary voltage which is equal to unit at the substation and decreases as we are moving far from the first substation and increases the more, we get closer to the next substation. The biggest voltage drop is identified between the second and the third substation with a value of 0.99 p.u system.

**Table 5:-** Change in total rail resistance due to total ground resistance.

| Rail resistance (RN) | Total Ground resistance (Rg) | | Equivalent rail Resistance (Req) | |
|---|---|---|---|---|
| | In terms of Percentage of RN (%) | Actual value (Ω) | Actual value (Ω) | Resistance per unit length (Ω/km) |
| 0.03372 | 25 | 0.00843 | 0.006744 | 0.004 |
| 0.03372 | 50 | 0.01686 | 0.01124 | 0.0066667 |
| 0.03372 | 75 | 0.02529 | 0.01445 | 0.008571 |
| 0.03372 | 100 | 0.03372 | 0.01686 | 0.01 |
| 0.03372 | 200 | 0.06744 | 0.2248 | 0.01333 |
| 0.03372 | 1000 | 0.3372 | 0.03065 | 0.018182 |
| 0.03372 | 6000 | 2.0232 | 0.33167 | 0.019672 |
| 0.03372 | 12000 | 4.0464 | 0.3344 | 0.19834 |
| 0.03372 | 80000 | 26.976 | 0.033678 | 0.01999 |

In the table5, the equivalent resistance keeps increasing as the total ground resistance increases.
The resistance per unit length of the DC Overhead Contact system for Addis Ababa Light Rail Transit is RL = 0.123Ω/km and the variation of the rail resistance due to different ground resistances is tabulated above.

**Table 6:-** The current, bus voltage and rail potential for different values of total ground resistance.

| Total ground Resistance (Rg) % of RN | V2 | I2 | Rail Potential - VN |
|---|---|---|---|
| 25 | 797.7354 | 891.7430 | 6.0139 |
| 50 | 798.6813 | 890.6869 | 10.0163 |

| | | | |
|---|---|---|---|
| 75 | 799.3531 | 889.9384 | 12.8587 |
| 100 | 799.8580 | 889.3766 | 14.9949 |
| 200 | 801.0315 | 888.0737 | 19.9589 |
| 1000 | 802.7418 | 886.1816 | 27.1927 |
| 6000 | 803.2567 | 885.6135 | 29.3701 |
| 12000 | 803.3127 | 885.5518 | 29.6070 |
| 80000 | 803.3687 | 885.4900 | 29.8438 |
| ∞ | 803.3722 | 885.4862 | 29.8586 |

The effect of rail potential on the bus voltage can be seen in the table above. The result is as expected theoretically. When the total ground resistance increases, Rg, increases, the rail potential also increases. When the rail potential increases, the bus voltages increase slightly, and the current through the rail deceases slightly. Even if the current through the rail decreases, the voltage increasing because the resistance of the rail is increasing.

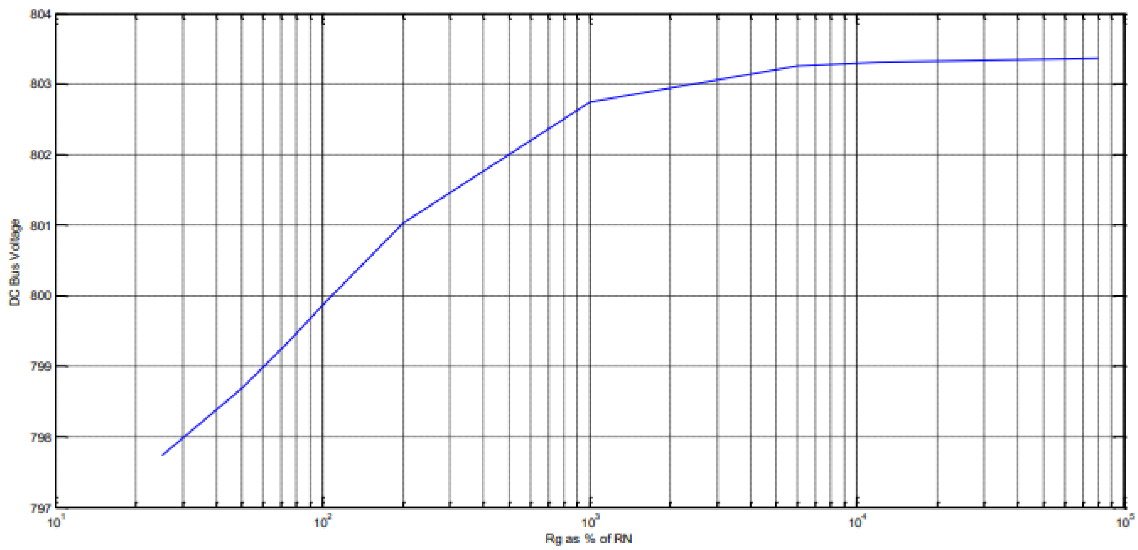



**Figure 13:-**Voltage vs total ground resistance graph in logarithmic scale.

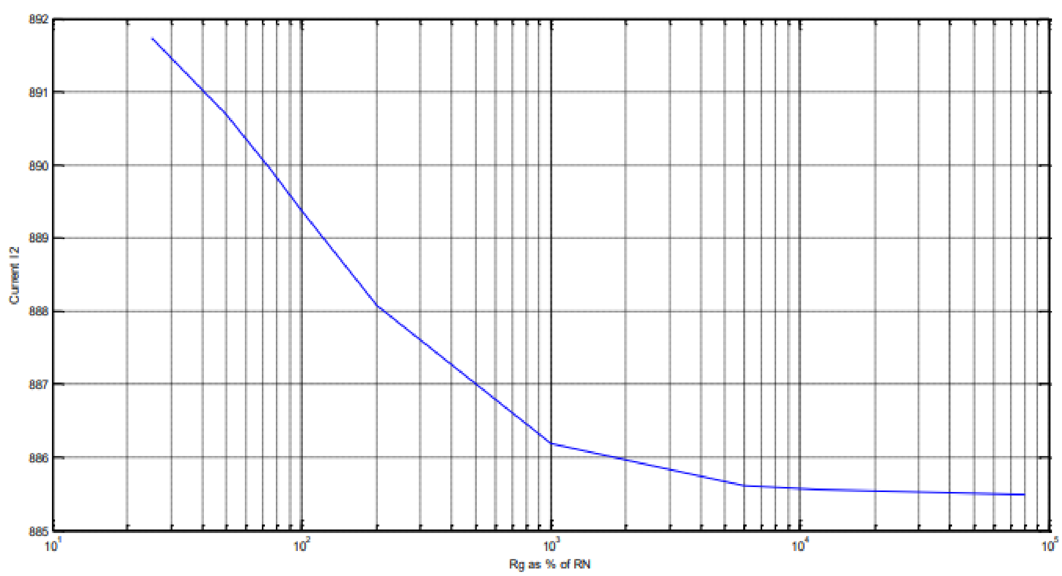



**Figure 14:-** Current versus total ground resistance graph in logarithmic scale.

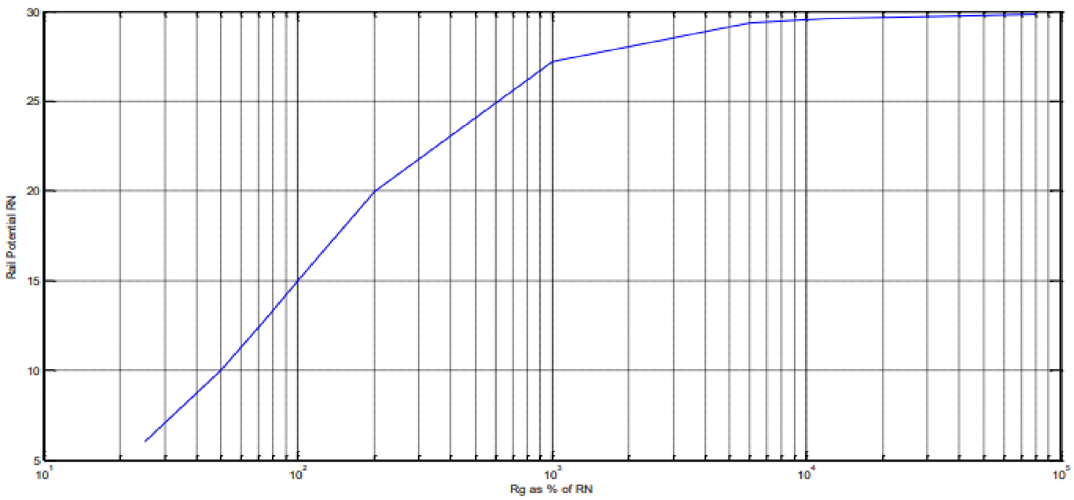



**Figure 15:-** Rail potential versus total ground resistance graph in logarithmic scale.

This result shows that we need an effective ground resistance in order to reduce the rail potential and bring it to a levelthat does not harmpeopleorotherlivingthings.Typically,conductorswithvaluesthatarecloseorlessthanthetotalrailresistance are more suitable for grounding at the station and the train in order to reduce the rail potential and reduce the risk relatedtoit.Whilechangingthevaluesofthegroundresistance,thechangeinthecurrentis7Ampereonly, andalso the change in voltage is also 7 volts only while the change in the rail potential is observed to be 23 volts. If the rail resistance is increased the change of the voltage will be critical. In addition, as it can be seen from the graphs (figure 13, 14 and 15) when the ground resistance is greater than the rail resistance, the change in rail voltage becomes small compare to it becoming smaller than the rail resistance, so good grounding should be done at the station and thetrain.

## Conclusion:-

When designing a DC railway system, and railway systems in general, the tractive effort and the power required to operate the train should be considered. For calculating the power consumption, the maximum load, the number of passengers, and the auxiliary power requirement should also be considered. It should be confirmed that the motive power required can be supplied by the motors on the train and under no condition even when considering the slopes and curves shall the power required be greater than the power that is to be supplied by the motors as this may cause accidents.

The other consideration while designing DC traction systems is the rail potential. Even if the rail resistance is very small, as it shares the current with the train, larger current may cause significant voltage drops. This voltage drop can cause harm to people and infrastructures around it. So, it should be limited with in a safe limit. As it was seen in the result, reducing the rail resistance does not interrupt the bus voltage and current significantly, but it is helpful in limiting the rail voltage. Most modem DC traction rail systems are totally floating systems. In such systems, the increase in rail voltages is a serious problem. Therefore, special precautions must be taken when running rails are used as the return current conductor and insulated from the ground. This is usually done using devices called rail potential control devices. This is also another method to reduce the rail potential.

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