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Redundant Systems with Repairable and Non-Repairable Failures

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Abstract

This paper studies two non-identical operative units and one cold standby unit with repairable and non-repairable failures. The system has one repairman for repair and replacement moreover; the system has some repairable failures states and some non-repairable failures states. In the case of the emergence of repairable failures states, the system will be repaired. When one non-repairable failure state appears, the system would never operate again. Thus, many classical reliability indices may become meaningless. For example, the steady state availability and failure frequency would become zero. We derive some new reliability indices of the system with repairable and non-repairable failures.

INTRODUCTION

Many systems have both repairable failures and non-repairable failures. For example, military equipment repairable failures that have been caused by its own breakdowns and damage as well as non-repairable failures that result from disappearance, destruction, capture, another ones that result from crash, explosion, loss of contact or controls and so on. The system that has both repairable failures and non-repairable failures called RANRF system. The repairable systems and the non-repairable systems are special cases of RANRF system. Which means the system will sooner or later become non-repairable failure (being absorbed) and stop operating. According on this concept, all reliability indices of a repairable system [1-8] may change. For example, the steady state availability and failure frequency may become zero. So some new reliability indices and new calculating methods should be introduced.

This paper is extended to [9]. The main idea of this paper is to derive a reliability model called RANRF system, which has some repairable failures and some non-repairable failures, and derive some reliability indices using the supplementary variable technique and the Laplace transform.

2. Assumptions

1. The system consists of two non-identical operative units and one cold standby unit
2. When an operative unit fails, immediately sent for repair by general time distribution. A repaired unit returns into operation
3. All failure rates are constant and the repair time distributions are arbitrary and different.
4. When both units fail, the repair is stopped and the standby unit replaces the last operative unit. Following the replacement, the repair process restarts again from the beginning.
5. When the standby unit fails, the system needs fast repairing and hence is repaired as a whole with a general distribution.
6. After repairing any one of the active units, it is sent into operation immediately and the operating standby unit returns into the cold standby state
7. The system would never operate again when attaining a non-repairable failure mode.
8. All random variables are independent
9. After the repairs, the unit is as good as new

3. Notation

λ_i	Constant failure rate of unit i, (i = 1, 2).
λ_3	Constant failure rate of the standby unit
λ_{ij}	The failure rate (with respect to non-repairable failure). When the system is in S_i (i=0,…,7), it can transfer to non-repairable failure mode with λ_{ij}
α	Constant replacement rate to replace the latest failed active units by the standby unit.
$P_i(t)$	Probability that the system is in state i at time t,
$P_i(t,x)$	Probability density, (with respect to repair time x) that the system is in state i (i=1, 2, 5, 6) at time t, and the unit under repair has an elapsed repair time x.
$P_i(t,y)$	Probability density, (with respect to repair time y) that the system is in state 7 at time t, and the unit under repair has an elapsed repair time y.
$\varphi_i(x),\ f_i(x)$	Repair rate and probability density function (p.d.f.) of repair time of a failed unit i (i = 1,2) and has an elapsed repair time of x. $f_i(x)=\varphi_i(x)\exp[-\int_0^x \varphi_i(u)du] \quad i = 1,2$
$\mu_0(y),\ h(y)$	Repair rate and p.d.f, of the system in state 7, and has an elapsed repair time of y. $h(y)=\mu_0(y)\exp[-\int_0^y \mu_0(u)du]$
$g^*(s)$	Laplace transform of a function g(t) Where $g^*(s)=\int_0^\infty g(t)\exp[-st]dt$

Let S(t) be the system state at time t, then we can get the possible states of the system are shown as follows:

State 0	unit 1 and 2, of the system are operating,
State 1	unit 1 is being repaired, and unit 2 is operating,
State 2	unit 2 is being repaired, and unit 1 is operating,
State 3	unit 2 fails, while unit 1 is being repaired,
State 4	unit 1 fails, while unit 2 is being repaired,
State 5	standby unit is operating, while unit 1 is being repaired and unit 2 is waiting for repair,
State 6	standby unit is operating, while unit 2 is being repaired and unit 1 is waiting for repair,
State 7	All units are down and the system is under repair,
State 8i	$i=1,...,j$,
State 9i	$i=1,...,k$,
State 10i	$i=1,...,L$,
State 11i	$i=1,...,m$,
State 12i	$i=1,...,n$,
State 13i	$i=1,...,p$,
State 14i	$i=1,...,r$,
State 15i	$i=1,...,q$: the states are non-repairable failures (absorption states)

The transition state of the system is described by Figure 1 according to a Markov chain, where the states 0, 1, 2, 5 and 6 are working states; while the states 3, 4 and 7 are failure states.

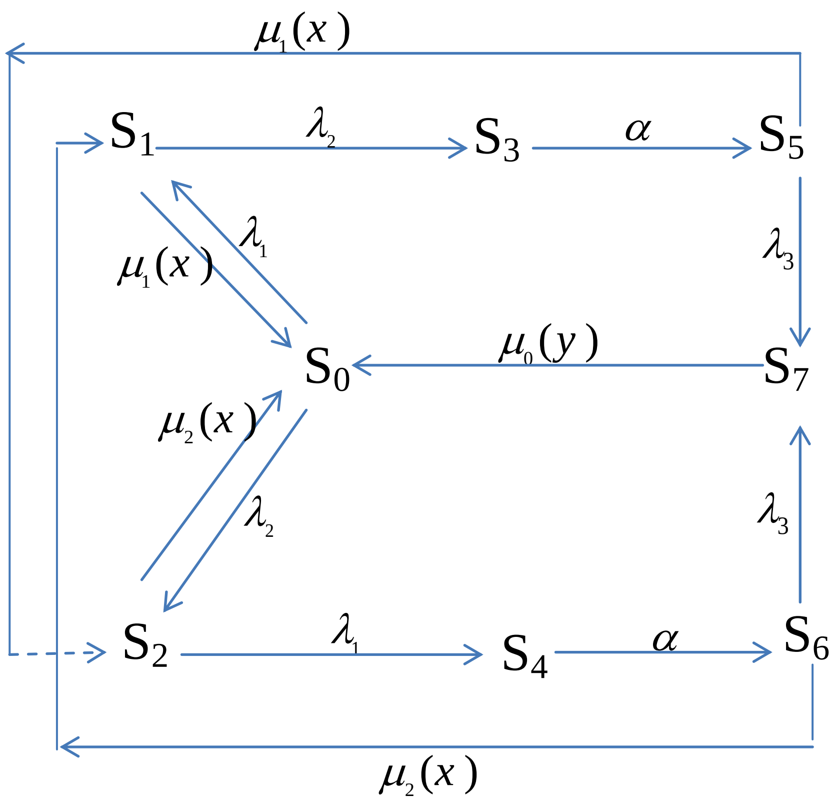

Fig.1 State transition diagram of model

4. The State Probabilities of the System

According to the assumptions of the system, we can get following differential equations for the system:

$$\left(\frac{d}{dt}+\sum_{i=1}^{2}\lambda_i+\sum_{i=1}^{j}\lambda_{8i}\right)P_0(t)=\sum_{i=1}^{2}\int_0^\infty \varphi_i(x)P_i^*(t,x)dx+\int_0^\infty \mu_0(y)P_7(t,y)dy \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\varphi_1(x)+\sum_{i=1}^{k}\lambda_{9i}\right)P_1(t,x)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\varphi_2(x)+\sum_{i=1}^{L}\lambda_{10i}\right)P_2(t,x)=0 \tag{3}$$

$$\left(\frac{d}{dt}+\alpha+\sum_{i=1}^{m}\lambda_{11i}\right)P_3(t)=\lambda_2P_1(t) \tag{4}$$

$$\left(\frac{d}{dt}+\alpha+\sum_{i=1}^{n}\lambda_{12i}\right)P_4(t)=\lambda_1P_2(t) \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_3+\varphi_1(x)+\sum_{i=1}^{p}\lambda_{13i}\right)P_5(t,x)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_3+\varphi_2(x)+\sum_{i=1}^{r}\lambda_{14i}\right)P_6(t,x)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)+\sum_{i=1}^{q}\lambda_{15i}\right)P_7(t,y)=0 \tag{8}$$

Their boundary conditions are:

$$P_1(t,0)=\lambda_1 P_0(t)+\int_0^{\infty}\varphi_2(x)P_6(t,x)dx \tag{9}$$

$$P_2(t,0)=\lambda_2 P_0(t)+\int_0^{\infty}\varphi_1(x)P_5(t,x)dx \tag{10}$$

$$P_5(t,0)=\alpha P_3(t) \tag{11}$$

$$P_6(t,0)=\alpha P_4(t) \tag{12}$$

$$P_7(t,0)=\lambda_3\sum_{i=5}^{6}P_i(t) \tag{13}$$

$$P_4(t,0)=P_5(t,0)=0 \tag{14}$$

Initial condition

$$P_0(0)=1,\ \ P_i(0,x)=0,\ \ x\neq 0,\ \ i=(1,2,4,6,7), P_i\ (0)=\ 0,\ i=(3,4) \tag{15}$$

$$P_i^*(s)=\int_0^{\infty}P_i^*(s,x)dx,\quad i=(1,2,5,6,7)\ \text{, we get} \tag{16}$$

Taking Laplace transforms of the set of equations (1-14) and using (15) and (16), we get:

$$\left(s+\sum_{i=1}^{2}\lambda_i+\sum_{i=1}^{j}\lambda_{8i}\right)P_0^*(s)=1+\sum_{i=1}^{2}\int_0^{\infty}\varphi_i(x)P_i^*(s,x)dx+\int_0^{\infty}\mu_0(y)P_7^*(s,y)dy \tag{17}$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_2+\varphi_1(x)+\sum_{i=1}^{k}\lambda_{9i}\right)P_1^*(s,x)=0 \tag{18}$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_1+\varphi_2(x)+\sum_{i=1}^{L}\lambda_{10i}\right)P_2^*(s,x)=0 \tag{19}$$

$$\left(s+\alpha+\sum_{i=1}^{m}\lambda_{11i}\right)P_3^*(s)=\lambda_2 P_1^*(s) \tag{20}$$

$$\left(s+\alpha+\sum_{i=1}^{n}\lambda_{12i}\right)P_4^*(s)=\lambda_1 P_2^*(s) \tag{21}$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_3+\varphi_1(x)+\sum_{i=1}^{p}\lambda_{13i}\right)P_5^*(s,x)=0 \tag{22}$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_3+\varphi_2(x)+\sum_{i=1}^{r}\lambda_{14i}\right)P_6^*(s,x)=0 \tag{23}$$

$$\left(s+\frac{\partial}{\partial y}+\mu_0(y)+\sum_{i=1}^{q}\lambda_{15i}\right)P_7^*(s,y)=0 \tag{24}$$

Their boundary conditions are:

$$P_1^*(s,0)=\lambda_1 P_0^*(s)+\int_0^{\infty}\varphi_2(x)P_6^*(s,x)dx \tag{25}$$

$$P_2^*(s,0)=\lambda_2 P_0^*(s)+\int_0^\infty \varphi_1(x)P_5^*(s,x)dx \quad (26)$$

$$P_5^*(s,0)=\alpha P_3^*(s) \quad (27)$$

$$P_6^*(s,0)=\alpha P_4^*(s) \quad (28)$$

$$P_7^*(s,0)=\lambda_3\sum_{i=5}^{6}P_i^*(s) \quad (29)$$

$$P_4^*(s,0)=P_5^*(s,0)=0 \quad (30)$$

Solving (17)-(24) with the help of (25)-(30), we get

$$P_0^*(s)=\frac{1}{D(s)} \quad (31)$$

$$P_1^*(s)=(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(A(s)+\lambda_1)(1-f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$/D(s)\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1$$
$$+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\} \quad (32)$$

$$P_2^*(s)=-\lambda_2\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))$$
$$/D(s)(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)$$
$$(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\} \quad (33)$$

$$P_3^*(s)=\lambda_2(A(s)+\lambda_1)(1-f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$/D(s)\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1$$
$$+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\} \quad (34)$$

$$P_4^*(s)=-\lambda_1\lambda_2\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))$$
$$/D(s)(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})$$

$$+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\} \tag{35}$$

$$P_5^*(s)=\alpha\lambda_2(A(s)+\lambda_1)(1-f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))(-1+f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i}))$$
$$/D(s)(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1$$
$$+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\} \tag{36}$$

$$P_6^*(s)=\alpha\lambda_1\lambda_2\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))$$
$$/D(s)(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})$$
$$(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\} \tag{37}$$

$$P_7^*(s)=\{\alpha\lambda_2\lambda_3(1-h^*(s+\sum_{i=1}^{q}\lambda_{15i}))/(s+\sum_{i=1}^{q}\lambda_{15i})D(s)\}\left\{\left\{(A(s)+\lambda_1)(1-f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))\right.\right.$$
$$(-1+f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i}))/(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})$$
$$\left.+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\right\}$$
$$+\left\{\lambda_1\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))\right.$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}$$
$$/(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})$$
$$\left.\left.(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}\right\}\right\} \tag{38}$$

Where

$$A(s)=\alpha\lambda_1\lambda_2(1-f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})$$
$$/(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i}) \tag{39}$$

$$D(s)=(s+\lambda_1+\lambda_2+\sum_{i=1}^{j}\lambda_{8i})-\left\{\left\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(A(s)+\lambda_1)\right.\right.$$
$$f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})+\lambda_2\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1$$
$$\left.+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})\right\}$$
$$/\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$\left.f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}\right\}$$
$$-\alpha\lambda_2\lambda_3h^*(s+\sum_{i=1}^{q}\lambda_{15i})\left\{\left\{\{(A(s)+\lambda_1)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))\right.\right.$$
$$(-1+f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i}))\}/\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)$$
$$\left.(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\right\}$$
$$+\left\{\{\lambda_1\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})-\alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))\right.$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}$$
$$/\{(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})$$
$$\left.\left.(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})+\alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\right\}\right\} \quad (40)$$

5. Reliability Indices

The ideas of reliability indices, we can get new reliability indices of the system as follows.

5.1 Time to Non-Repairable Failure

There is a period of time from starting operation of the system to entering non-repairable failure. The time T is defined as the time to non- repairable failure. We define T is a random variable, its c.d.f. G(t) is called the cumulative distribution function of the time to non-repairable failure,

$$G(t)=P\{T<t\}=1-P\{T>t\},$$

where $P\{T>t)=\overline{G}(t)$ is the probability that the system has not entered non-repairable failure at time t. From the assumptions of the system, we get

$$\overline{G}(t)=P_0(t)+\sum_{i=1}^{2}\int_0^\infty P_i(t,x)dx+\sum_{i=3}^{4}P_i(t)+\sum_{i=5}^{6}\int_0^\infty P_i(t,x)dx+\int_0^\infty P_7(t,y)dy \quad (41)$$

Taking the Laplace transforms on both sides of (41) and using equation (16) and using (31)-(40), we can get $\overline{G}^*(s)$ as follows

$$\overline{G}^*(s) = P_0^*(s) + \sum_{i=1}^{2}\int_0^\infty P_i^*(s,x)dx + \sum_{i=3}^{4} P_i^*(s) + \sum_{i=5}^{6}\int_0^\infty P_i^*(s,x)dx + \int_0^\infty P_7^*(s,y)dy$$

$$\overline{G}^*(s) = \frac{1}{D(s)}\Bigg\{1 + B(s) + C(s) + 1/\{(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$
$$(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})(s+\sum_{i=1}^{q}\lambda_{15i})\}\Bigg\{\lambda_2 B(s)(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})$$
$$\Bigg\{\alpha\lambda_3(-1+h^*(s+\sum_{i=1}^{q}\lambda_{15i}))(-1+f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})) + (s+\sum_{i=1}^{q}\lambda_{15i})$$
$$\{(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i}) + \alpha - \alpha f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\Bigg\} + \lambda_1 C(s)(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})$$
$$(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\Bigg\{\alpha\lambda_3(-1+h^*(s+\sum_{i=1}^{q}\lambda_{15i}))(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))$$
$$+(s+\sum_{i=1}^{q}\lambda_{15i})\{(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}) + \alpha - \alpha f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\Bigg\}\Bigg\}\Bigg\} \quad (42)$$

Where

$$B(s) = (s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(A(s)+\lambda_1)(1-f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))/\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})$$
$$(s+\alpha+\sum_{i=1}^{m}\lambda_{11i}) + \alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\} \quad (43)$$

$$C(s) = -\lambda_2\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})(s+\alpha+\sum_{i=1}^{m}\lambda_{11i}) - \alpha\lambda_1(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))$$
$$f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\}(-1+f_2^*(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i}))/(s+\lambda_1+\sum_{i=1}^{L}\lambda_{10i})\{(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i})$$
$$(s+\alpha+\sum_{i=1}^{m}\lambda_{11i}) + \alpha A(s)(-1+f_1^*(s+\lambda_2+\sum_{i=1}^{k}\lambda_{9i}))f_1^*(s+\lambda_3\sum_{i=1}^{p}\lambda_{13i})\} \quad (44)$$

The expected value of T is defined as the mean time to non-repairable failure (MTTNRF). Denoting E (T) as MTTNRF, we have $E(T) = \overline{G}^*(0)$ as follow

$$E(T) = \overline{G}^*(0) = \frac{1}{D(0)}\Bigg\{1 + B(0) + C(0) + 1/\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$
$$(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})(\sum_{i=1}^{q}\lambda_{15i})\}\Bigg\{\lambda_2 B(0)(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\Bigg\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))$$
$$(-1+f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})) + (\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}) + \alpha - \alpha f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\Bigg\}$$
$$+\lambda_1 C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\Bigg\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))$$

$$+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\alpha-\alpha f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\Big\}\Big\}\Big\} \tag{45}$$

5.2 Availability

The system is in operation if and only if it is in the states S_i for $i=0,\ 1,\ 2,\ 5,\ 6$. The probability $Av(t)$ that the system is in operation at time t is defined as the instantaneous availability of the system at time t. We obtain

$$Av(t)=P_0(t)+\sum_{i=1}^{2}\int_0^{\infty}P_i(t,x)dx+\sum_{i=5}^{6}\int_0^{\infty}P_i(t,x)dx \tag{46}$$

Taking the Laplace transforms of the equation (46) and using equation (16) and using (31)-(33), (36) and (37) we can get $Av^*(s)$

$$Av^*(s)=P_0^*(s)+\sum_{i=1}^{2}\int_0^{\infty}P_i^*(s,x)dx+\sum_{i=5}^{6}\int_0^{\infty}P_i^*(s,x)dx$$

$$Av^*(s)=\Big\{1/D(s)(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\Big\}$$
$$\Big\{\mathrm{B(s)}(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$
$$+\lambda_2(\alpha-\alpha f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))\}+(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$
$$\Big\{(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+C(s)\{(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})$$
$$-\alpha\lambda_1(-1+f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}\Big\}\Big\} \tag{47}$$

The mean operation time T_1 from starting operation of the system to entering non-repairable failure is defined as the mean operation time (MOT) of the system. From the definition and use the limiting theorem of the Laplace transforms. From (47), we get

$$T_1=\int_0^{\infty}Av(t)dt=\lim_{s\to\infty}\int_0^{t}Av(t)dt=\lim_{s\to 0}sL[\int_0^{t}Av(t)dt]=\lim_{s\to 0}Av^*(s) \tag{48}$$

The mean availability of the system, Av, is defined as $Av=T_1/E(T)$, which may be used instead of the steady state availability of a common. Using (45) and (48), we obtain:

$$Av=\Big\{(\sum_{i=1}^{q}\lambda_{15i})\Big\{(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$
$$(1+B(0)+C(0))-\alpha\lambda_2B(0)(-1+f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))\}-\alpha\lambda_1C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})$$
$$(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(-1+f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\Big\}\Big\}/\Big\{(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})$$
$$\Big\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(\sum_{i=1}^{q}\lambda_{15i})(1+B(0)+C(0))+\lambda_2B(0)\Big\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))$$

$$(-1+f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})+\alpha-\alpha f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\Big\}\Big\}$$

$$+\lambda_1 C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\Big\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))$$

$$+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\alpha-\alpha f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\Big\}\Big\} \quad (49)$$

5.3 Replacement and Repairable Failure Frequency

The system suffers replacement and repairable failure if and only if it transfers from state $i,(i=1,\ 2)$ into state $i,(i=3,\ 4)$ and state $i,(i=5,\ 6)$ into state 7 respectively. Denoting $W(t)$ as the instantaneous replacement and repairable failure frequency of the system at time t, we obtain

$$W(t)=\int_0^\infty \lambda_2 P_1(t,x)dx+\int_0^\infty \lambda_1 P_2(t,x)dx+\sum_{i=5}^{6}\int_0^\infty \lambda_3 P_i(t,x)dx \quad (50)$$

Taking the Laplace transforms of the equation (50) and using equation (16) and using (32), (33), (36) and (37) we can get $W^*(s)$, we have

$$W^*(s)=\int_0^\infty \lambda_2 P_1^*(s,x)dx+\int_0^\infty \lambda_1 P_2^*(s,x)dx+\sum_{i=5}^{6}\int_0^\infty \lambda_3 P_i^*(s,x)dx$$

$$W^*(s)=\Big\{\lambda_2 B(s)(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$

$$+\lambda_3(\alpha-\alpha f_1^*(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))\}+\lambda_1 C(s)(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$

$$\{(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\lambda_3(\alpha-\alpha f_2^*(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}\Big\}$$

$$/\Big\{D(S)(s+\alpha+\sum_{i=1}^{m}\lambda_{11i})(s+\alpha+\sum_{i=1}^{n}\lambda_{12i})(s+\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(s+\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\Big\} \quad (51)$$

Let M represent the mean numbers which the system suffers replacement and repairable failure from starting operation of the system to entering non-repairable failure. From (51), we get

$$M=\int_0^\infty W(t)dt=\lim_{s\to\infty}\int_0^t W(t)dt=\lim_{s\to 0} sL[\int_0^t W(t)dt]=\lim_{s\to 0} W^*(s) \quad (52)$$

We define $W=M/E(T)$ as the mean replacement and repairable failure frequency of the system, which may be used instead of the steady state failure frequency of a common repairable system, then using (45) and (52), we get the mean replacement and repairable failure frequency of the system as follow

$$W=M/E(t)=\Big\{(\sum_{i=1}^{q}\lambda_{15i})\Big\{\lambda_2 B(0)(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$

$$+\lambda_3(\alpha-\alpha f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))\}+\lambda_1 C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$

$$\{(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\lambda_3(\alpha-\alpha f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\}\Big\}\Big\}$$

$$/\left\{(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\left\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(\sum_{i=1}^{q}\lambda_{15i})\right.\right.$$

$$(1+B(0)+C(0))+\lambda_2 B(0)\left\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))\right.$$

$$\left.\left.+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})+\alpha-\alpha f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\right\}\right\}+\lambda_1 C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})$$

$$(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\left\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))\right.$$

$$\left.\left.+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\alpha-\alpha f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\right\}\right\} \quad (53)$$

5.4 Repair Frequency of the Unit

The unit 1 is repaired if and only if the system transfers from state S_0 into state S_1. We define $W_1(t)$ as the instantaneous repair frequency of unit 1 at time t, we obtain

$$W_1(t)=\lambda_1 P_0(t) \quad (54)$$

Taking the Laplace transforms of equation (54) and using (31), we have

$$W_1^*(s)=\lambda_1 P_0^*(s)=\lambda_1/D(s) \quad (55)$$

Let M_1 represent the mean repair numbers of unit 1 from starting operation of the system to entering non-repairable failure. Applying probability analysis and from (55), we get

$$M_1=\int_0^\infty W_1(t)dt=\lim_{s\to\infty}\int_0^t W_1(t)dt=\lim_{s\to 0} sL[\int_0^t W_1(t)dt]=\lim_{s\to 0} W_1^*(s)=\lambda_1/D(0) \quad (56)$$

Denote the quotient $W_1=M_1/E(t)$ as the mean repair frequency of unit 1. From equations (45) and (56), we obtain the mean repair frequency of unit 1

$$W_1=M_1/E(t)=\lambda_1/\left\{1+B(0)+C(0)+1/\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\alpha+\sum_{i=1}^{n}\lambda_{12i})\right.$$

$$(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})(\sum_{i=1}^{q}\lambda_{15i})\}\left\{\lambda_2 B(0)(\alpha+\sum_{i=1}^{n}\lambda_{12i})(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\right.$$

$$\left\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\right.$$

$$\left.+\alpha-\alpha f_1^*(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\right\}+\lambda_1 C(0)(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\left\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))\right.$$

$$\left.\left.\left.(-1+f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\alpha-\alpha f_2^*(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\right\}\right\}\right\} \quad (57)$$

Similarly, we can obtain the instantaneous repair frequency of unit 2 at time t

$$W_2(t)=\lambda_2 P_0(t) \quad (58)$$

Taking the Laplace transforms of (58) and (31), we have

$$W_2^*(s)=\lambda_2 P_0^*(s)=\lambda_2/D(s) \quad (59)$$

We deduce the mean repair numbers of unit 2 from starting operation of the system to entering non-repairable failure from (59) as follow:

$$M_2 = \lim_{s\to 0} W_2^*(s) = \lambda_2 / D(0) \tag{60}$$

We define $W_2 = M_2 / E(t)$ as the mean repair frequency of unit 2. From (45) and (60), we get the mean repair frequency of unit 2

$$W_2 = M_2/E(t) = \lambda_2 / \Big\{1 + B(0) + C(0) + 1/\{(\alpha + \sum_{i=1}^{m}\lambda_{11i})(\alpha + \sum_{i=1}^{n}\lambda_{12i})$$
$$(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i})(\sum_{i=1}^{q}\lambda_{15i})\}\Big\{\lambda_2 B(0)(\alpha + \sum_{i=1}^{n}\lambda_{12i})(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i})$$
$$\Big\{\alpha\lambda_3(-1 + h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1 + f_1^*(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})) + (\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})$$
$$+\alpha - \alpha f_1^*(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})\}\Big\} + \lambda_1 C(0)(\alpha + \sum_{i=1}^{m}\lambda_{11i})(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})\Big\{\alpha\lambda_3(-1 + h^*(\sum_{i=1}^{q}\lambda_{15i}))$$
$$(-1 + f_2^*(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i})) + (\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i}) + \alpha - \alpha f_2^*(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i})\}\Big\}\Big\}\Big\} \tag{61}$$

Similarly, we can obtain the instantaneous repair frequency of standby unit at time t

$$W_3(t) = \int_0^\infty \lambda_3 P_5(t,x)dx + \int_0^\infty \lambda_3 P_6(t,x)dx \tag{62}$$

Taking the Laplace transforms of (62) and using (16), (36) and (37), we have

$$W_3^*(s) = \int_0^\infty \lambda_3 P_5^*(s,x)dx + \int_0^\infty \lambda_3 P_6^*(s,x)dx$$

$$W_3^*(s) = -\alpha\lambda_3\Big\{\lambda_2 B(s)(s + \alpha + \sum_{i=1}^{n}\lambda_{12i})(s + \lambda_3 + \sum_{i=1}^{r}\lambda_{14i})(-1 + f_1^*(s + \lambda_3 + \sum_{i=1}^{p}\lambda_{13i}))$$
$$+\lambda_1 C(s)(s + \alpha + \sum_{i=1}^{m}\lambda_{11i})(s + \lambda_3 + \sum_{i=1}^{p}\lambda_{13i})(-1 + f_2^*(s + \lambda_3 + \sum_{i=1}^{r}\lambda_{14i})\Big\}$$
$$/\Big\{D(S)(s + \alpha + \sum_{i=1}^{m}\lambda_{11i})(s + \alpha + \sum_{i=1}^{n}\lambda_{12i})(s + \lambda_3 + \sum_{i=1}^{p}\lambda_{13i})(s + \lambda_3 + \sum_{i=1}^{r}\lambda_{14i})\Big\} \tag{63}$$

We can get the mean repair numbers of standby unit from starting operation of the system to entering non-repairable failure from (63) as follow:

$$M_3 = \lim_{s\to 0} W_3^*(s) \tag{64}$$

Denote the quotient $W_3 = M_3 / E(t)$ as the mean repair frequency of standby unit. From (45) and (64), we get the mean repair frequency of standby unit.

$$W_3 = M_3/E(t) = -\Big\{\alpha\lambda_3(\sum_{i=1}^{q}\lambda_{15i})\{\lambda_2(\alpha + \sum_{i=1}^{n}\lambda_{12i})(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i})B(0)(-1 + f_1^*(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i}))$$
$$+\lambda_1(\alpha + \sum_{i=1}^{m}\lambda_{11i})(\lambda_3 + \sum_{i=1}^{p}\lambda_{13i})C(0)(-1 + f_2^*(\lambda_3 + \sum_{i=1}^{r}\lambda_{14i}))\}\Big\} / \Big\{(\alpha + \sum_{i=1}^{n}\lambda_{12i})$$

$$(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\Big\{(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})(\sum_{i=1}^{q}\lambda_{15i})(1+B(0)+C(0))+\lambda_2 B(0)$$

$$\Big\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))(-1+f_1^{\ *}(\lambda_3+\sum_{i=1}^{p}\lambda_{13i}))+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})$$

$$+\alpha-\alpha f_1^{\ *}(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})\}\Big\}\Big\}+\lambda_1(\alpha+\sum_{i=1}^{m}\lambda_{11i})(\lambda_3+\sum_{i=1}^{p}\lambda_{13i})C(0)\Big\{\alpha\lambda_3(-1+h^*(\sum_{i=1}^{q}\lambda_{15i}))$$

$$(-1+f_2^{\ *}(\lambda_3+\sum_{i=1}^{r}\lambda_{14i}))+(\sum_{i=1}^{q}\lambda_{15i})\{(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})+\alpha-\alpha f_2^{\ *}(\lambda_3+\sum_{i=1}^{r}\lambda_{14i})\}\Big\}\Big\} \quad (65)$$

6. Conclusions

This paper studied some reliability indices of a RANRF system that consists of both repairable and non-repairable failures. Using the supplementary variable technique, probability analysis, definite integral and the Laplace transform, we derived corresponding calculating methods. In addition, those reliability indices are different from the corresponding indices of both common repairable systems and common non-repairable systems.

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