Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI: 10.21474/IJAR01/10924
DOI URL: http://dx.doi.org/10.21474/IJAR01/10924

RESEARCH ARTICLE

ANALYSIS OF RHEOLOGICAL MODELS OF NON- NEWTONIAN FLUIDS

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Manuscript Info

Manuscript History
Received: 08 March 2020
Final Accepted: 10 April 2020
Published: May 2020

Key words:-
Non-Newtonian, Visco-Elastic, Visco-Inelastic, Constitutive Equation

Abstract

In this paper, an attempt has been made to analyse the development of non-Newtonian fluid from the classical theory of Newtonian liquid. Various rheological models of constitutive equations of different kinds of non-Newtonian liquid have been discussed. Development of the theory of non-Newtonian fluid overcoming the limitations of classical theory of perfect fluid has been discussed in detail. Classifications of visco-elastic as well as visco-inelastic fluids are also analysed.

Introduction:-

A fluid is a substance which can flow. All liquids and gases are fluids. Euler gave his famous equations of fluid flow for perfect fluids in 1755 in his paper "General Principles of the Motion of Fluids", fluid flow for perfect fluids in 1755. This can be considered a turning point in the way of development of the research in this field, making giant step altering the very nature of engineering applications. This follows the development of perfect fluid. The assumption of linearity of relation between the stress and the rate of strain tensors and the normal direction of stress vector on the plane surface in contact led to the concept of perfect fluids. Euler's partial differential equations were non-linear but solutions could be obtained in some special cases like flows past circular and elliptic cylinders and past spheres and ellipsoids. This perfect or ideal fluid theory gave excellent results for certain classes of problems such as wave formation and tidal motions until the appearance of D'Alembert's paradox.

The journey of science is the non-tiring pursuance of perfection. A magnificent breakthrough came in 1904 with the mathematician Prandtl proposing his assumption of boundary layer. These have revolutionized the study of motions of ships and lift and drag on airplane wings. Many mathematicians has begun to study and analyse different problems using models of different kinds of non- Newtonian fluids. Charyulu, V. N. and Ram, M. S [1] studied the flow of a visco-elastic second order fluid through porous medium.

Classical Theory and Its Inadequacy:

Newton [2] in 1687 suggested the constitutive equations of isotropic, viscous incompressible fluid as

$$\sigma_{ij} = -p\delta_{ij} + 2\mu . e_{ij}\, , e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right), \qquad (2.1)$$

which is a linear equation.

Here, σ_{ij} is stress tensor, p is indeterminate hydrostatic pressure, δ_{ij} is kronecker delta, μ is the coefficient of viscosity, v_i are velocity components.

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It could give a good approximation to that of ordinary mobile liquids like water, glycerin, honey and many thin oils and could explain the phenomena of lift, skin friction, form drag, separation and secondary flows etc. But it fails to explain a number of phenomena observed in case of a large number of fluids with technological and industrial importance such as thick oils, paints, lubricants, starch solution, rubber toluene solutions, colloids and suspensions and so on. These fluids show a distinct departure from Newtonian hypothesis. Such types of fluids are defined as non-Newtonian fluids.

Fortunately, the never ending race towards the perfection has led to the development of different tools for coping with challenge of explaining the phenomena of non-Newtonian fluids whose flow properties differ in any way from those of Newtonian fluids.

The most commonly defining property of non-Newtonian fluid is that the viscosity is dependent on shear rate or shear rate history. Fluid resists its gradual deformation by the shear or tensile stress. This ability is measured by the viscosity. Many commonly found examples of non-Newtonian fluids are many salt solutions, ketchup, custard, starch suspensions, molten polymer, blood, commonly used paints, shampoo etc.

The linearity of relations between the stress and shear rate is different in non-Newtonian fluid. This may even be time depended. As such the coefficient of viscosity cannot be constant in non-Newtonian fluids. Non-Newtonian fluids are studied with the tools of different rheological models relating stress and strain rate tensors.

Visco-elastic Fluids:

The very name suggests that such type of fluids possess a certain degree of elasticity. In the state of motion, certain amount of energy is stored up as strain energy while a part of energy is lost due to viscous dissipation. Following the removal of the stress it recovers its original state and sometimes a reverse flow is also possible. The strain is responsible for these phenomena. Hence, in the case of visco-elastic fluid flow the strain cannot be neglected, however small it may be. The concept of "memory" may be mentioned at this point. This relates to the measure of elasticity of the fluid. During the flow of such a fluid, its natural state changes constantly and its continuous endeavor is to attain the instantaneous state of the deformed state, but it does never succeed completely. This lag is the so called "memory" of the fluid.

Some of the visco-elastic fluids are analyzed below.

Oldroyd Fluid

The constitutive equation as proposed by Oldroyd [3, 4, 5] of this fluid is

$$\sigma_{ij} = \tau_{ij} - Pg_{ij}, \qquad (3.1)$$

where σ_{ij} is the stress tensor,

g_{ij} is the metric tensor,

P is an isotropic pressure,

τ^{ij} is given by the relation

$$\tau^{ij} + \lambda_1 \frac{D\tau^{ij}}{Dt} + \mu_0 E^{ij}\tau_k^k + \nu_1 E^{kl}\tau_{kl} g^{ij} = 2\eta_0 \left[E^{ij} + \lambda_2 \frac{DE^{ij}}{Dt} + \nu_2 E^{kl} E_{kl} g^{ij} \right], \qquad (3.2)$$

Where, $E_{ij} = \frac{1}{2}\left(U_{j,i} + U_{i,j}\right)$, (3.3)

U_i being the velocity vector,

η_0 a constant having the dimensions of viscosity,

λ_1, λ_2 (relaxation time and retardation time parameters respectively),

μ_0, ν_1 and ν_2 are constants having the dimensions of time.

$\frac{D}{Dt}$ is the convected derivative .

Rivlin-Ericksen Fluids:

The constitutive equation for Rivlin-Ericksen fluids from purely phenomenological considerations is

$$\sigma = -pI + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2 \qquad (3.4)$$

where p is an arbitrary hydrostatic pressure and ϕ 's are constants.

Matrices A_1 and A_2 are defined by

$$A_{ij}^{(1)} = \left(v_{i,j} + v_{j,i}\right)$$

$$A_{ij}^{(2)} = \frac{\partial A_{ij}^{(1)}}{\partial t} + v_p A_{ij,p}^{(1)} + A_{ip}^{(1)} v_{p,j} + A_{pj}^{(1)} v_{p,i}$$

v_p being velocity vector.

It is customary to call ϕ_1 the coefficient of ordinary viscosity. ϕ_2 the coefficient of viscoelasticity, and ϕ_3 the coefficient of cross viscosity.

Maxwell fluids:

The constitutive equation for this class of fluids, has been proposed by Maxwell as

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\tau^{ij} = 2\mu e^{ij} \qquad (3.5)$$

where λ is the relaxation time for the stress.

Walters Fluid (Model B$^{/}$):

The constitutive equation of Walters liquid (Model B$^{/}$) is given by

$$\sigma_{ij} = -p g_{ik} + \tau_{ij} \qquad (3.6)$$

$$\tau^{ij}(x,t) = 2\int_0^{\infty} \varphi\left(t - t^{/}\right) \frac{dx^j}{dx^{im}} \cdot \frac{dx^j}{dx^{ir}} e^{(1)mr}\left(x^{/}, t^{/}\right) dt^{/} \qquad (3.7)$$

where σ_{ij} is the stress tensor,

p is an arbitrary isotropic pressure,

g_{ij} is the metric tensor of fixed coordinate system,

$x^i, x^{/i}$ the position at the time $t^{/}$ of the element which is instantaneously at the point x^i at time t,

$e_{ij}^{(1)}$ the rate of strain tensor and

$$\varphi\left(t - t^{/}\right) = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-\left(t - t^{/}\right)/\tau} d\tau \qquad (3.8)$$

$N(\tau)$ being the distributive function of the relaxation time τ .

It has been shown by Walters [6,7] that in case of liquids with short memories (i.e. short relaxation times), the equation of state can be simplified to

$$\tau^{ij} = 2\eta_0 e^{(1)ij} - 2K_0 \frac{\delta}{\delta t} e^{(1)ij} \qquad (3.9)$$

where $\eta_0 \left(= \int_0^{\infty} N(\tau) d\tau\right)$ is the limiting viscosity at small rate of shear,

$$K_0 = \int_0^{\infty} \tau N(\tau) d\tau,$$

and $\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor quantity, which for any contra variant tensor b^{ij} is given by,

$$\frac{\delta}{\delta t} b^{ij} = \frac{\partial b^{ij}}{\partial t} + v^m \frac{\partial b^{ij}}{\partial x^m} - \frac{\partial v^j}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mj} \qquad (3.10)$$

where v_i is the velocity vector.

Visco-inelastic Fluids:

Let us now come to the visco-inelastic fluid. These types of fluids are generally isotropic and homogeneous. When subjected to a shear, the resultant stress depends only on the rate of shear. Of course there can be seen diverse behavior in response to stress applied. Some of the rheological models of visco-inelastic fluids are

1. **Power law Fluids:**

A Power law fluid is characterized by the rheological equation

$$\sigma_{ij} = k\left|\left(\sum_{m=1}^{3}\sum_{l=1}^{3} e_{ml}e_{lm}\right)^{\frac{1}{2}}\right|^{n-1} e_{ij}\,, \qquad (4.1)$$

Here k is the consistency index and n is the flow behavior index respectively.

If $n < 1$, the fluid is called pseudo plastic power law fluid. In case of such fluid the apparent viscosity and shear rate have an inverse relationship.

For $n > 1$, the apparent viscosity is directly proportional to shear rate and such type of fluid is called dilatant power law fluid.

Some of the visco-inelastic fluids are Reiner-Rivlin Fluid with constitutive equation

$$\sigma_{ij} = -p\delta_{ij} + 2\mu e_{ij} + 2\mu_c e_{ik}e_{kj}\,,$$

μ_c being the coefficient of cross viscosity

Bingham Plastics, with constitutive equation

$$\sigma = \pm\sigma_0 + \mu.e\,,$$

σ_0 being the yield value which is equal to zero in Newtonian fluid.

Without going inside, let us mention some other types of non-Newtonian fluids as

2. Dipolar Fluids
3. Anisotropic Fluids
4. Fluid With Microstructure
5. Heat Conducting Nematic Liquid Crystals

Conclusion:-

The above mentioned models are most vital tools for analysing various problems in fluid dynamics. The main characteristic of such rheological models is the presence of parameters or indexes defining the nature of the fluid. If we put the value of this parameter equal to zero, we get the results for Newtonian fluid.Many commonly found examples of non-Newtonian fluids are many salt solutions, ketchup, custard, starch suspensions, molten polymer, blood, commonly used paints, shampoo etc. Framing and solving different problems involving such types of fluids has wide scope of applications in different fields like chemical industries, paint industries, medical treatments, aviation and maritime industries.

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