Journal Homepage: - www.journalijar.com
INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)
ISSN NO. 2320-5407

Article DOI: 10.21474/IJAR01/5673
DOI URL: http://dx.doi.org/10.21474/IJAR01/5673

RESEARCH ARTICLE

STRUCTURAL LINEARITY AND NON-LINEARITY.

Vaibhv P Karnik, Rishabh Madhani and Vivek Gupta.
Department of Civil Engineering, TCET Mumbai.

Manuscript Info

Manuscript History

Received: 20 August 2017
Final Accepted: 22 September 2017
Published: October 2017

Abstract

Linear and Non Linear behavior is the most important behavior of the structure to be analyzed while designing any type of structure. Linear and Non Linear behavior as the name says is the behavior of a structure when the set of loads are acting on a structure which includes both live loads and dead loads of a structure and the analysis of this behavior of structure is known as linear and non linear analysis of structures. The main function of analysis of this structure is to check the stability of structure when the loads starts acting on it, even it gives ultimate stress i.e. the stress at which the structure collapse. So with help of this we can also find design stress with suitable factor of safety.

Introduction:-

This paper shows the detailed explanation of linear and non linear analysis of various structures with their detail explanation. This paper explains various types of non linearity such as material and geometric non linearity.

It also provides detail explanation of non linearity of concrete-steel composite structures and cable structures. The paper also describes about the non linearity of seismic structures.

Linear Analysis

Linear analysis is a method of proportional analysis which states that “when load is applied on a structure, it obeys Hooke’s Law within elastic limit and strain developed in a structure is directly proportional to the stress applied.” This behavior of a structure in elastic limit is known as linear behavior of a structure and the analysis of this behavior of structure within elastic limit is known as linear analysis.

Let say, for example: The deflection ‘D’ is generated due to a moment ‘M’ then deflection of ‘3D’ will be generated when the moment applied will be ‘3M’. This behavior of structures is known as linear behavior. All principles of superposition are valid in linear analysis.

One simple example of superposition is: The deflection caused by the dead load of a beam is 1 inch and the deflection caused by the live load applied on the beam is 0.4 inch then according to superposition principle the total deflection will be the sum of deflection caused by dead load and live load which is 1.4 inch.

Since the structure follows a linear pattern it indicates that it follow Hooke’s Law within elastic limit and the equation for the straight line is given by ‘Y=mX’. From mechanics point of view ‘Y’ is replaced by ‘Stress’ (σ), X is replaced by ‘Strain’ (e) and the slope of line ‘m’ is replaced by ‘Modulus of Elasticity’ (E). The equation turns to σ=E.e

Corresponding Author:- Vaibhv P Karnik.
Address:- Department of Civil Engineering, TCET Mumbai.

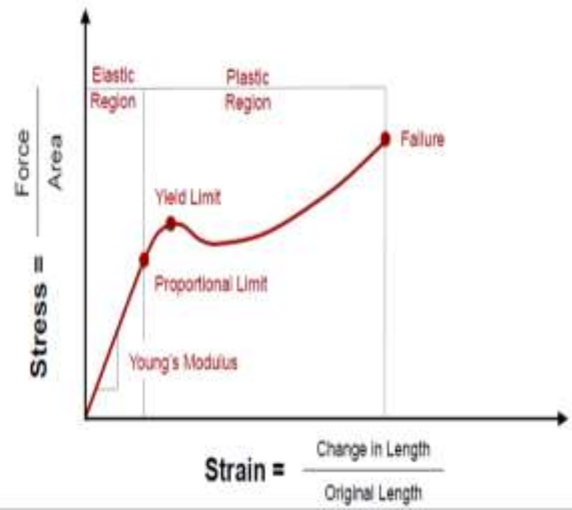


Figure 1:- Graph of Hooke's Law

Figure 1 shows that the structure behaves linearly till the yield point.

For different engineering materials the linear behavior changes, this can be seen in Figure 2.

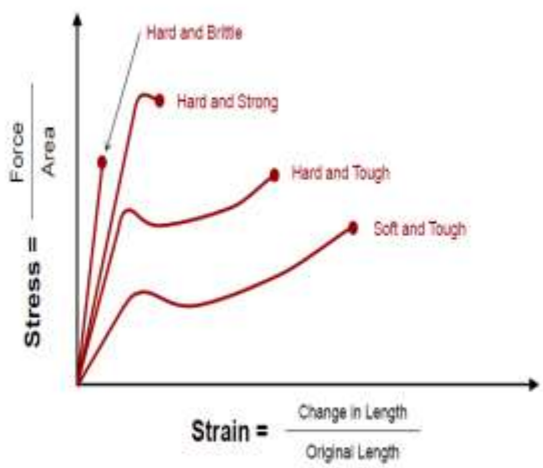


Figure2:- Linear Behavior for different materials.

Non-Linear Analysis:-

When the load is applied on a structure it does not obeys Hooke' Law, the strain developed due to stress induced varies non-linearly, this occurs above the yield point. This behavior of the structure is known as non-linear behavior and its analysis is called as non-linear analysis of structure.

There are two types of Nonlinearity observed in structures which are as follows:-

Material Nonlinearity:-

Materials tend to lose their linear behavior as soon as they reach a zone which is beyond their yield strength. Many properties of the materials undergo changes when they enter this zone of which some of them are:

Permanent deformations:-

When load is applied to a material and then when it is unloaded the material will not regain its original shape or position but will remain in its deformed shape. This phenomenon in which material does not regain its original shape is called as permanent deformation. For e.g. - When a plastic bag is stretched after some time when you release the bag one can see permanent stretch marks on it. This is a simple example of permanent deformation.

Cracking:-
This phenomenon occurs in linear analysis s well which is mostly observed in case of concrete, during the calculations of seismic design we consider the reduced stiffness of members which is nothing but the assumed value. In case of nonlinear analysis generally the cracking of concrete is monitored such that the concrete will crack and eventually the members will start to lose their stiffness.

Beam Rotations:-
When a moment greater than the capacity of a beam is applied to it, the beam forms a plastic hinge instead of rotating and thus starts dissipating energy. This is included in material nonlinearity but it is called as backbone curve in case of beams. Only the capacity of the member is taken into account in case of linear design.

Energy Dissipation:-
Energy dissipation in linear analysis is in the form of strain energy and in the form of inelastic energy in case of nonlinear analysis in addition to strain energy dissipation.

A few in general observations were made in nonlinear analysis of which some of them are- A member will lose its stiffness when the load applied is larger than the capacity of the member. The applied load will cross the elastic limit of the member which will lead to further hardening or cracking. Hence, a structure is loaded until it reaches thee nonlinear stage further we check the check the condition of the structure by checking the amount of cracks developed. In order to achieve accuracy the same process is repeated several times. Due to this repetition the time taken for nonlinear analysis is more than linear analysis.

Linear analysis fails to provide the complete information about the failure of a structure in case of seismic activities like an Earthquake. Higher accuracy up to 90% is achieved with the help mathematical models. But the results obtained by the analysis through linear dynamics are not so accurate. For e.g.- If a beam is subjected to earthquake shakes the beam will experience some load which will be limited. The beam is therefore designed in such a way that it can handle such limited force. But in reality if a beam is to an actual earthquake we will find that the beam is actually shattered. Such shattering of beam on increased load is not acceptable.

That is why nonlinear analysis is preferable over linear analysis for more accurate and in depth results.

Geometric Nonlinearity:-
Based on the concept of force follower approach the most famous geometric nonlinearity is called as P-Delta analysis. As the deflection increases it indicates the presence of some additional forces which are generated by P-Delta effects and it is called as “Geometric Nonlinearity”. According to force follower analysis as soon as a member loses its stability the deformed member is followed by the force which creates further instability. P-Delta analysis is a complex method whose effects may be adverse if neglected. The meaning of P-Delta analysis originates from load and lateral deformation which stands for P and Delta accordingly. P-Delta analysis is very significant because it helps to check the capacity of columns so that they don’t fail if additional moments are applied along with axial loads.

Although P-Delta analysis have many adverse effects which are mostly observed in case of tall buildings. Whenever a building is affected by an earthquake it leads to its deformation. The structure is already in its inelastic zone with concrete cracking and furthermore the deformation is huge too. This causes the structure to lose its stiffness. The force generated at the top & bottom of the column because of P-delta moments also called as P-Delta shear leads to an increase in demand for lateral shear resistance of the structural system. This increase in demand causes an increase in shear demands of an earthquake. This suggests that if we provided sufficient shear resistance and did not consider the P-Delta demands then the building.

Analysis of Concrete-Steel Composite:-
Steel-concrete composite framed buildings come under highly efficient structural systems due to their properties like stiffness, strength and ductility. These systems help enhance seismic performance even though their applications is reduced due to lack of experimental data and rules of design, especially joints. The present in this field depends upon seismic design and testing of typical steel and concrete composite multi-story moment resisting frame up to 4 stories. After the design of frame through linear analysis, a number of nonlinear analysis tests are performed by lumped plasticity model by introducing models for beam-column and plastic joint Since, plastic rotation capacity is

not properly defined, the formulations for steel structure are discussed and the results are then defined in terms of q-factor which are in accordance to the international codes.

To evaluate the performance of structures in seismic areas static nonlinear analysis is a very important too although this method is not applicable in all cases. In case of composite beams which have a very high asymmetric behavior of sagging or hogging and also which gets affected by width and beam slab connection the calculations become very complex because there is very little information available about the property like plastic rotation capacity i.e. ductility.

The problem of energy dissipation can be solved with the help of capacity design procedure of composite frames. With the help of international standards a strategy to find out dissipative capacity of frames can be done by finding the plasticity at ends of beams instead of columns. This can be done by using full strength beam-column joints ensuring full plasticization in beams and not joints. Studies have found out that ductility and dissipation capacity are superior to their behavior factors indicating they could have higher ductility in beams and at base column. The columns ductility is determined on the basis of its type i.e. fully encased, concrete filled or partially encased. The beams are generally made by coupling between a steel profile in lower section and a slab on the upper section. The rotational capacity can be varied by varying the ductility of the concrete i.e. the hogging moment can be limited through buckling. This can also be termed as the threshold value of plastic deformation or critical stress.

The Design of the Composite Frame:-

Design of a multi-storey building is extracted from analyzed frame used as offices. The building is regular in both plan and elevation. Dimensions of plan are: 31m and 24m in longitudinal and transverse direction respectively. And the total area covered is 744 sqm. The height from ground plan and first floor is 14.5m and 4m for a total of 4 stories.

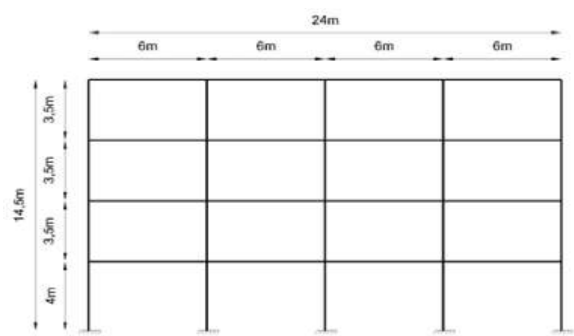


Figure3:- Analyzed frame

Material	Class	Mechanical features				
		f_{ck}	f_{yk}	f_u	γ_m	E
		MPa	MPa	MPa	(-)	MPa
Conc	C20/25	20			1.50	29962
Steel bar	B450C		450	540	1.15	210000
Steel	S275		275	430	1.1	210000

Table1:- Mechanical characteristics of the materials

Columns used were partially encased. The beams contained IPE profiles and RCC slabs having a height of 120mm. The design was made according to Euro code 8 rules by applying the concept of capacity design by column-beam strength in bending and shearing criteria. A reference was made by using a seismicity of medium-high level under PGA value of 0.08g for service limit state and 0.25g which is the ultimate state. The performance for ultimate limit state and service limit state were checked using software.

The moment-curvature relationship of beams and columns composite sections:

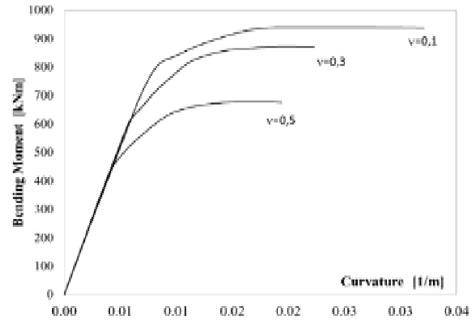


Figure4:- The moment-curvature relationship of columns

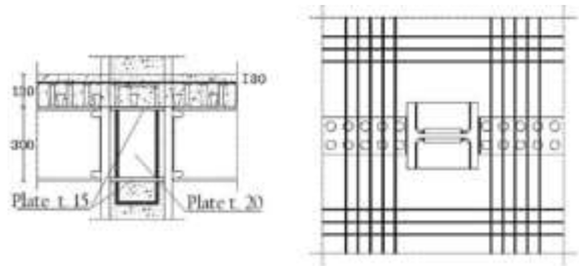
The moment-curvature relationships of composite sections were tailored for the various sections of beams and columns assuming the Bernoulli hypothesis (the plane section remains plane) between the various components (concrete, structural steel, reinforced steel) and nonlinear constitutive relationships for the materials.

For concrete, the Mander non-linear model (Mander et al, 1988) was implemented since it could take into account the confinement degree of concrete and in particular that of the composite columns between the flanges of steel profile. The ultimate strain of the confined concrete was determined by the expression reported in (Scott et al., 1998), that gives a value of 0.03 Instead, for unconfined concrete the ultimate strain of 0.5% was established (Scott et al., 1998). The stress-strain of concrete in tension was assumed linear-brittle. For a compression cylindrical characteristic strength, f_{ck}, of 20 MPa, according to the standards formulas of Eurocode 2 (2004), gives a tensile strength f_{ctm} = 2.21 MPa and an elastic modulus E = 29962 MPa, assumed equal in compression and tension.

The steel grade of the reinforcement is B450C (f_{yk} = 450 MPa), and for this one an elastic perfectly plastic (EPP) stress-strain curve was adopted. Also for the steel of beams and columns an EPP stress-strain curve was adopted, with an ultimate strain of 1% and 2%, respectively in compression and in tension; the yield characteristic stress value, f_{sy} , is 275 MPa, both for the beams and columns. The limit value of deformation in compression was established to control, with an indirect way, the occurrence of local buckling phenomena (Kemp, 1985). This value seems too much detrimental especially in the case of the columns. For this reason the $\square_{su}$ value for columns was based on the formulation of Elnashai et al. (Elnashai et al. 1998) obtaining about 3.5%.

Comparison of the global nonlinear response considering rigid or deformable joints:-

In this paragraph are reported the results of non-linear static analyses concerning the designed steel- concrete composite frame, considering or not the joint deformability. In particular, two types of full strength joints were considered: a typical welded join and flanged one. Both joints where first modeled using a sophisticated approach based on the component methods (Amadio et al. 2011) where the components of the joint are schematized as non-linear springs. In the obtained macro-model the identified components are: web panel in shear (spring1), column web in tension and/or compression (springs 2), T-stub elements in tension (springs 3), beam-to-slab connection in shear (spring 4), the mechanism 1 and 2 provided by Eurocode 4 and the slab-column interaction (springs 5, 6, 7, 8). Then, the deformability of the connection was introduced into the model through an NLlink (Multi linear plastic) element of the SAP2000, which describes the Moment-rotation curve of the connection itself evaluated by the components method. Using the aforementioned moment-rotation laws, two frames were modeled taking into account the deformability of welded or flanged joints. It is worth to notice as the overall stiffness of the frame with rigid joints is higher (about 25%) than that of the frame where the deformability of the steel and concrete components of the joints are considered. Furthermore, the joints deformability causes an enhancement of the yielding and ultimate displacement of about 22% for triangular distribution and 25% for that constant one. In particular, at the SLS the maximum inter-story drift, in the case of modal distribution of seismic forces, for the frame with rigid joints results 0.0054 and becomes 0.0085 in the case of deformable joints , overcoming greatly, in the last case, the limits imposed by codes for infill panels rigidly connected. For what concerns the level of base shear, about the same value is reached for the two models.



Behavior of Cable Structures:-

In order to determine the accurate behavior of the cable structure, nonlinear analysis is necessary. There are no explicit references available to make differences between linear & nonlinear structures. In the paper, the explicit difference between linear & nonlinear structure has been discussed.

Cable structures are supposed to withstand loads without slackening, in practice. Thus, some pretension will remain in all the structural members, under the maximum probable loads.

Engineering structures are considered to behave linearly. Analysis of engineering structures is based upon the linear relationship between forces & displacement. The nonlinear responses are the exception where the linear relationship between the displacement & forces are not described linearly behavior of structure. The nonlinearity of structures is due to:-

Nonlinear behavior of material.

Nonlinear geometric behavior.

But in behavior of cable structures geometric behavior is considered because it is dominating property in cable structures.

Structure Examples:-

The Biconcave Cable Structure:-

It is plane truss which consist of vertical hangers and span of length 70.00 m. It is constructed of ten 7.00m panels. Its height is 2.00 m in the center of the span and the total height of the truss is 11.35m. It is considered that cross-section of the cable is circular and the sectional areas of 27,8256 cm2 , 20,0862 cm2 and 0,5969 cm2 for the sagging, hogging and hanger cables. The modulus of elasticity of the cables is 16500 kN/cm2. The truss is supported by 4 pin-joints and has a total of 36 degrees of freedom.

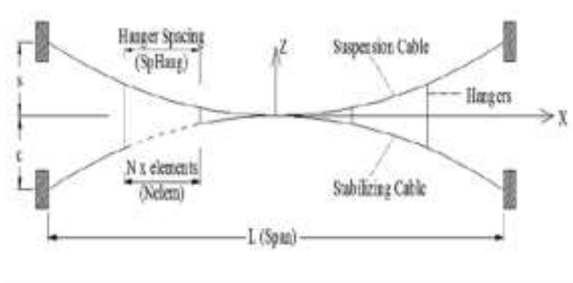


Figure5:- Biconcave Cable Structure.

Hyperbolic Paraboloid Cable Structure:-

Hyperbolic Paraboloid is a structure of dimension 35,052 x 35,053 m. It is a structure used in Airports Buildings. It has shape function:

$$y = \frac{5.334}{17.526^2} z^2 - \frac{3.81}{17.526^2} x^2$$

It consists of seven cables in each way in equal spacing. The cables have EA=210,312 MN and the horizontal component of cable prestress tension is 200.17 kN for cables in the x-direction and 142.97 kN for cables in the y-direction. The structure has 75 degrees of freedom.

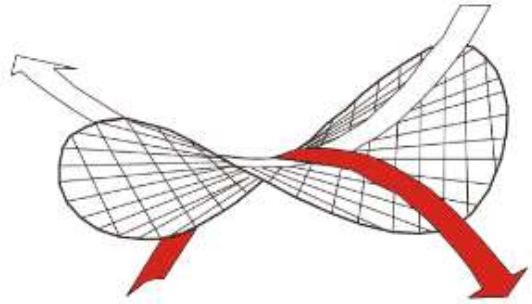


Figure6:- Hyperbolic Paraboloid Cable Structure.

Results:-

The Biconcave Cable Structure:-

Evaluation is based on two loading patterns. Firstly, at different levels of pretension same loads were loaded. Secondly, loads were increased to maximum value that doesn't slacken the structure for each case of pretensioning. The pretensioning of the cable truss was varied from 20% to 50%. There were 15 load step considered. The graphs of the above are shown below.

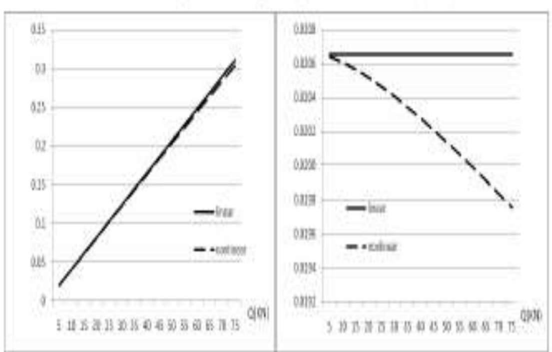


Figure7:- Load-displacement curve.

Pretension	Loading case	%*
20%	5	0.463
	10	1.157
	15	1.973
30%	5	0.606
	10	1.338
	15	2.146
40%	5	0.674
	10	1.426
	15	2.229
50%	5	0.708
	10	1.469
	15	2.264

Table2:- Displacement for loading pattern.

Hyperbolic Paraboloid Cable Structure:-

Same loading patterns were considered for two pretension cases for Hyperbolic Paraboloid Cables. The first case is the one found in literature which corresponds to a pretension of 10% and the second is a pretension of 5%. If initial equilibrium configuration of structure are considered for measurement of displacements two patterns of linear/nonlinear load displacements curves can be plotted. The curves are shown below.

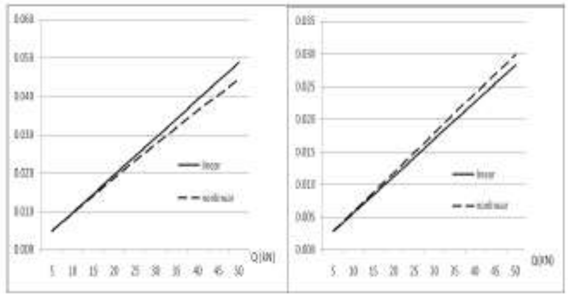


Figure8:- Load-displacements curves.

The Table below shows the difference in percents between displacement and linear/nonlinear analysis.

Pretension	Loading pattern	
	1	2
5%	0-6.04	0-15.7
10%	0-6.04	0.29-11.37

Table3:- Results for the hyperbolic Paraboloid.

Non-Linear Seismic Response of Structure:-

Structures, when are designed, they are designed for the forces, which are very less as compared to designed earthquake forces. Therefore, when an earthquake occurs or when the structure is met with an earthquake, due to ground motion it undergoes inelastic deformation. The structures may not be collapsed but the damage done to the structure by the earthquake is beyond repair. Reinforced concrete cement structural systems are made ductile by providing reinforcing steel (As per IS 13920-1993). Structures which are sufficiently ductile undergo a large deformation in inelastic region. Time history analysis of different single degree of freedom & multiple degree of freedom structure which possess nonlinearity is required to be performed. The possible or true behavior of such structures can be understood; by such analysis of time history results of the above mentioned analysis will help in predicting whether the structure will collapse or did not collapse at all. Some structural systems possess linear inertia, damping & restoring force. Such structures are analyzed by linear method. Structural system which possesses any or all three reactive forces (i.e. Inertia, damping & restoring forces) having nonlinear responses namely displacement, velocity & acceleration, a set of nonlinear differential equation is formed. The nonlinear differential equations are solved to obtain the responses. The stiffness nonlinearity deals with geometric & material nonlinearity. For material nonlinearity, restoring action shows hysteric behavior under cycling loading where no such behavior is observed in geometric nonlinearity.

Dynamic problems associated to structural controls, offshore structures & aerodynamics of structures, damping nonlinearity is faced. Damping nonlinearity mostly is of non hysteretic type.

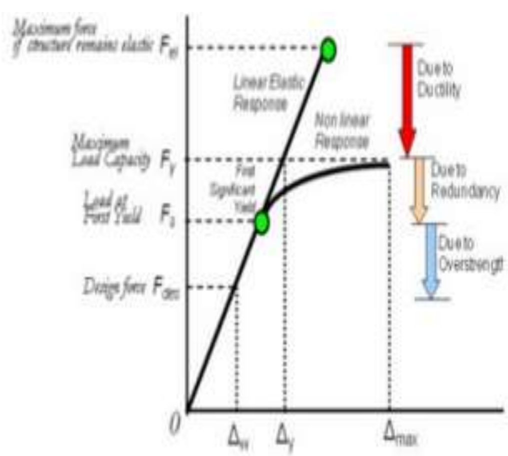


Figure8:- Graph between Total Horizontal Force and Roof Displacement.

SI No.	Structure	Importance Factor
1	Important service and community buildings, such as hospitals; schools; monumental structures; emergency buildings like telephone exchange, television stations, radio stations, railway stations, tire station buildings~ large community halls like cinemas, assembly halls and subway stations, power stations	1.5 19
2	All other buildings	1.0

Figure9:- Factors affecting Response of Structure.

Conclusion:-

In reference to this paper we can come up to a conclusion that linear and non linear behavior of a structure is very important to study while developing any type of structure. From this paper we got to know different aspects of linear and non linearity.

References:-

1. Non-linear behaviour of steel-concrete composite moment resisting frames marisa pecce university of sannio - engineering department benevento, italy.
2. Comparison Between the Linear and Nonlinear Responses of Cable Structures I – Static Loading By -Adrian Pintea1.