RESEARCH ARTICLE

CORRELATION BETWEEN MAXILLARY ARCH PERIMETER OF PRE AND POST ORTHODONTIC STUDY MODELS USING RAMANUJAN'S EQUATION FOR PERIMETER OF ELLIPSE.

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Manuscript Info

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Abstract

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Introduction: The most common problem encountered in borderline case is whether to extract or to expand the dental arch. The prediction of increase or decrease in arch perimeter by expansion or protraction or retroclination of incisors is a very much decisive. **Materials and Method:** 25 pairs of pre and post orthodontically treated study models were evaluated and correlated for measured arch perimeter with predicted arch length obtained by applying Ramanujan's equation for perimeter of ellipse. Linear and circumferential measurements were done directly on mid-buccal surfaces. **Results**: Ramanujan's equation had a high level of correlation (0.99) when comparing the measured perimeter of the maxillary arch with the calculated arch perimeter with 1.28% error. **Conclusion**: The ellipse is an accurate geometric shape that best fits the maxillary arch. The increase and decrease in the arch-length after expansion, proclination or retroclination of teeth can also be very well predicted.

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Introduction:-

The most important assessment for any orthodontist in his clinical diagnosis of dental mal-occlusion is the presence or absence of tooth crowding or spacing. Space requirement in the arches is a very crucial factor in deciding whether to extract or not to extract the teeth. Various treatment options to treat the arch-length discrepancies (ALD) are by extraction of teeth,[1] expansion of the arches,[2] proclination of the incisors,[3] or by reduction of the inter-proximal surfaces or distalization of the posterior segment.[4] The treatment modality should aim at improvement of functional efficiency, structural balance and aesthetic harmony.

A non-extraction orthodontic treatment plan incorporating dental arch expansion is often initiated in patients who exhibit minimal crowding or who would benefit from increased lip-support afforded by incisor advancement.[5] Decreased treatment time and retention of sound teeth are advantageous while perceived instability is one of the disadvantage when treated with expansion. However, without accurate prediction of the arch perimeter to be gained

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when proclining or expanding the arch in a borderline extraction patient, the orthodontist may underestimate the space gained and erroneously prescribe extractions that will then require excess space closure or, conversely, expect to gain more space and be unable to resolve the ALD.[6]

Dental arch-form is an important element in orthodontic treatment and retention. During orthodontic treatment, excessive tooth movement over the basal bone may lead to periodontal complications and an unstable treatment effect. In 1925, Lundstrom put forth the "apical base theory" to explain the boundaries of the expanding dental arch. He proposed that the supporting bones are not changed by orthodontic tooth movement or masticatory forces, and that their expansion is limited by the apical base bone.[7] Tweed CH[8] and Begg PR[9] also found strong evidence for the limitation of dental arch expansion. Prediction of the arch perimeter is an essential component when planning comprehensive orthodontic treatment modality.

Different amounts of arch perimeter changes in the canine and molar region have been proposed for the same amount of expansion. Ricketts et al suggested guidelines state that each millimeter of canine expansion provides for a 1mm increase in arch perimeter, and that 1 mm of molar expansion increased the perimeter by 0.25 mm.[6] In a study performed by Adkins et al, it was concluded that changes in premolar arch-width were approximately 0.7 times the premolar expansion.[10] When the canine width and incisor positions were held constant, an initial 1mm increase in molar width produced approximately 0.27 mm increase in perimeter, the second millimeter produced an increase of 0.31 mm, and the fifth millimeter of molar width increase was related to a perimeter increase of 0.41 mm.[11] Hnat et al found the canine: molar expansion ratio to be 1.25:1 as canine-to-molar width expansion ratios are a function of the point of application of the expansion force relative to the center of resistance of the dento-maxillary complex.[12]

Numerous studies have shown a wide range of readings which are very difficult to interpret when we try to attempt the prediction of arch-perimeter which is gained by either proclination of anteriors or by expansion of posteriors. A clear understanding of the arch-form is necessary for predicting the arch-perimeter; hence various studies have reported 'normal' dental arches which approximate certain geometric curves.[13]
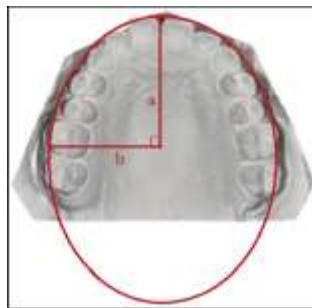
Most recent studies have concluded that ellipse is the best geometric figure for describing the arch-form of both the maxillary and mandibular dental arches. An ellipse is a plane curve that results from the intersection of a cone by a plane in a way that produces a closed curve. Several investigators have performed "endless series" of calculations for determining the exact perimeter of an ellipse. However, the most adequate and approximate formulation; widely considered to be the most accurate was formulated by Srinivasan Ramanujan in 1914.

The primary objective of this study was to quantitatively evaluate the effectiveness of Ramanujan's equation for predicting the perimeter of an ellipse which can be gained either by proclining the incisors or expansion of the molars and secondarily to mathematically correlate it to the maxillary arch-perimeter in pre and post-treatment models. Hopefully, the results of this study will help the orthodontist to plan the treatment of borderline extraction cases in a better way.

Materials and Method:-
The formulation of the adequate approximation by Srinivasan Ramanujan requires two values- *a* and *b,* i.e. semi-major and semi-minor axis [half of major axis and minor axis] respectively. The perimeter of an ellipse; [14]

$$P = \pi(a+b)\{1 + (3h/(10 - \sqrt{(4-3h)}))\}$$

Where $h = (a-b)^2/(a+b)^2$

The dental arch-form can be correlated to the perimeter of an ellipse by measurements of the semi-major axis *a* and semi-minor axis *b*. A line *l* is drawn to connect the mid-buccal surface of disto-buccal cusp of maxillary first molars on both sides. The perpendicular distance from this line to labial surface of maxillary central incisors is the semi-major axis *a;* while the semi-minor axis *b* is the half of the line *l* from mid-buccal surfaces of the disto-buccal cusps of maxillary first molars.

The correlation of the arch-form to a geometric shape curve allows adjustments of variables to aid in arch-perimeter prediction dynamically and also permits its representation in static form.

The samples for this study include the pre-treatment and post-treatment study models of the maxillary arch of the patients who underwent orthodontic treatment in Pt. Deendayal Upadhyay Dental College, Solapur, Maharashtra. 25 pre-treatment study models of non-extraction cases treated either by expansion of molars, proclining or retroclining the incisors were selected. The study models of patients who had full complement of teeth from first molar to first molar with crowding, spacing and rotations were included in the study. The sample consisted of maxillary arch study-models of subjects ranging in age from 11 to 35 years with mean age of 18 years.

A battery operated electronic digital caliper was used for recording the linear measurements on the study models. The measurements were taken from disto-buccal cusps of the maxillary first molars on each side. Semi-major axis *a* was the perpendicular distance from the line *l* to the labial surface of the maxillary central incisors measured using a metal ruler and the caliper. Semi-minor axis *b* was obtained by dividing line *l* into two parts. A small vertical line was marked from the disto-buccal cusp on the buccal surface of the maxillary first molar using a 0.3mm lead pencil (Staedler- Mars Micro Carbon).

The arch-perimeter was directly measured on the study models from the vertical line marked on the disto-buccal cusp of maxillary first molars with a 0.010inch stainless steel ligature wire that contacted the buccal surface of each tooth. The wire was marked at disto-buccal cusps with a marking pencil and then it was straightened and laid flat on a graph paper. The markings on the wire were then transferred on the graph paper and were measured with the caliper.

To evaluate the accuracy and reproducibility, 10 models were randomly selected and re-measured 1week after the initial measurements and analysis. The reliability of the recorded measurements was evaluated by Dahlberg's formula.[15] The data obtained from the measurements was then inserted into Ramanujan's equation for perimeter of an ellipse. The value *a* was the perpendicular distance from the labial surface of maxillary central incisors to the line bisecting the disto-buccal cusps of maxillary first molars. The linear measurement *b* was the distance from mid-buccal surfaces of disto-buccal cusps of the first molars divided into half.

Pearson correlation test was done to compare the values obtained from the calculations with Ramanujan's equation for perimeter of an ellipse with the values obtained by direct measurement of the study models from disto-buccal cusps of maxillary first molars using 0.010 inch stainless steel ligature wire that contacted the buccal surfaces of each tooth.

Table 1:- Measured and calculated values of pre and post-treatment models along with the % error
Rx-treatment, Meas.-measured, Cal.-calculated, %- percentage]

Sample	a [U1-6]		b [1/2 l]		Pre-Rx		Post-Rx		% Error	
	Pre-Rx	Post-Rx	Pre-Rx	Post-Rx	Meas.	Cal.	Meas.	Cal.	Pre-Rx	Post-Rx
1	29	32	25	26	81	83.37	96	97.2	2.86	1.23
2	39	35	30	29	108	107.3	101	99.7	0.64	1.28
3	37	33	25.5	28	101	99.32	98	97.3	1.29	0.71
4	39	35	29.5	28	106	104.2	100	99.4	1.69	0.6
5	36	34	27.5	26.5	103	101.4	97	96.1	1.5	0.92
6	37	34	27	28	99	100.7	101	103.1	1.68	1.84
7	30	32	27	26	94	91.7	96	94.2	2.44	1.87
8	38	36	28	28.5	106	105.4	105	104.7	0.56	0.28
9	36	34	27.5	27	103	102.5	100	99.2	0.48	0.8

| 10 | 31 | 33 | 29.5 | 28.5 | 101 | 99.3 | 105 | 104.3 | 2.27 | 0.66 |
11	34	32	28	27	109	107.6	105	103.7	1.28	1.23
12	36	31	28	28.5	104	100.2	95	93.8	3.65	1.26
13	36	34	28	27.5	102	101.1	94	93.4	0.88	0.63
14	35	33	27.5	28	103	101.4	104	103.6	1.55	0.38
15	27	29	27	27.5	89	87.2	88	86.8	2.02	1.36
16	46	44	30	29.5	122	119.8	119	117.3	1.80	1.42
17	36	33	26.5	27	96	94.7	95	94.2	1.35	0.84
18	39	37	28.5	27.5	98	97.4	96	94.3	0.61	1.77
19	43	41	29	28.25	111	109.7	108	107.2	1.17	0.74
20	31	32	27	28	99	97.8	98.5	97.2	1.21	1.31
21	35	33.5	27	26	102	100.7	99.5	98.7	1.27	0.80
22	39	37	26	27.75	103	101.5	101	100.5	1.45	1.48
23	33	32	27	26	98	97.4	95	94.3	0.61	0.73
24	38	35	28	26.5	101	99.8	99	98.3	1.18	0.70
25	41	39.5	30.5	29.5	115	112.7	112	110.9	2.0	1.78

Results:-

The results of Pearson correlation co-efficient (r) show high correlation of 0.990 and 0.991 respectively between measured and calculated values of arch perimeter for pre and post treatment study models. The level of significance (p) is highly significant; also 1.28% error was encountered when measured maxillary arch perimeter was compared with the calculated values after formulating in Ramanujan's equation for perimeter of ellipse.

	Groups	Mean	Standard Deviation	R	p
Pre-treatment	Measured	102.1600	8.00354	0.990	**.000***
	Calculated	100.9676	7.50900		
Post-treatment	Measured	100.3200	6.38990	0.991	**.000***
	Calculated	99.5760	6.37340		

Table 2:- Mean and SD values for pre and post treatment models, P*= highly significant]

Graph 1and 2 show pre and post-treatment measured and calculated values.
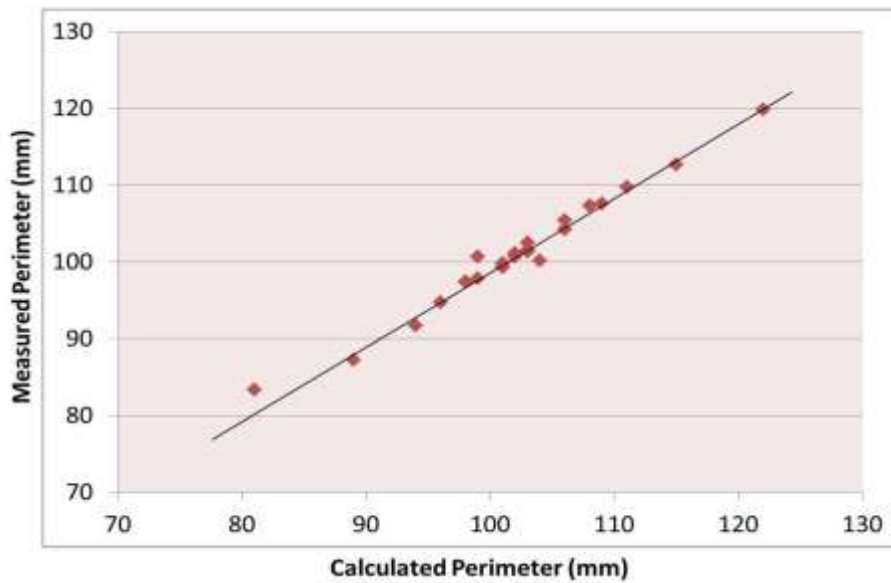
Graph 1:- Measured and Calculated pre-treatment values
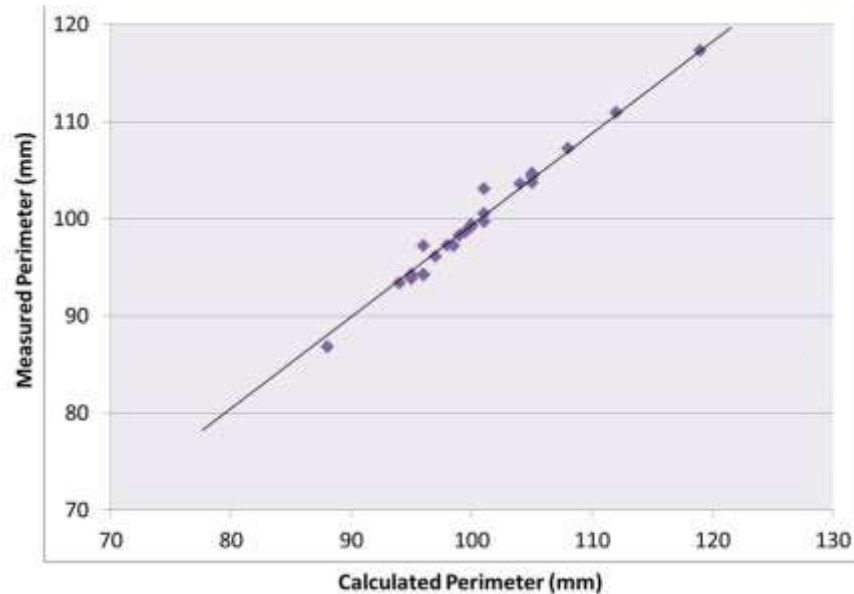
Graph 2:- Measured and Calculated post-treatment values

It can be evaluated that inter-molar expansion is about 0.684mm for every millimeter of increase in arch length. 1.035mm increase in arch length is seen for every millimeter of incisor advancement. Every 1 mm of incisor retraction reduces the arch length by 0.661mm. Arch length decreases by 0.27mm for every 1mm reduction of inter-molar width.

Discussion:-

Since ages, whatever treatment plan is decided for mal-occlusion of a particular patient, the basic aim to achieve an ideal arch-form at the end remains the same. Whether we expand or contract the dental arch, mesialize or distalize the posterior teeth or retract or protract the incisor teeth the arch perimeter changes.[15]

The primary purpose of this study was to quantitatively evaluate the effectiveness of Ramanujan's equation for predicting the perimeter of an ellipse and secondarily to mathematically correlate it to the maxillary arch-perimeter in pre and post-treatment models.

Various claims were made since the start of the 20th century regarding the fit of the ideal geometric curves to dental arch-forms. Most researchers recommended that the ideal fit for dental arch-form is some form of an ellipse, parabola or caternary curve.[12] Table 2 depicts the authors and their recommended arch-forms.

[Table 2. Authors and their recommended arch-forms with area of best-fit]

Sr. No.	Author	Form of Ideal Curve	Area of fit
1	Angle EH (1906-07)	Parabola	Middle of the dental arch
2	Ramanujan S (1914)	Ellipse	Average fit to dental arch
3	Williams PN (1917)	Circle	Incisor tips only
4	Stanton FL (1922)	Ellipse/ Parabola	Buccal cusps & incisal edges
5	Izard G (1927)	Ellipse in 75% Parabola in 20% U shaped in 5%	Middle curve of arch
6	McConnail & Shurr (1949)	Caternary curve	Average fit to dental arch
7	Wheeler RC (1950)	Parabola	Anterior of the arch
8	Sicher H (1952	Ellipse Parabola	Best fit to upper dental arch Best fit to lower dental arch
9	Lu KH (1964)	Polynomial equation	Middle curve

10	Mills & Hamilton (1965)	Parabola	-
11	Brader AC (1972)	Tri-focal ellipse	Middle curve
12	Herren P et al (1973)	Interpolation of small lines	To fit middle curve
13	Ackerman & Musich (1973)	Caternary curve	Average fit to dental arch
14	Hechter FJ (1978)	Parabola	Middle curve of arch
15	Begole (1979)	Cubic inter-polarity spline	-
16	Rudge SJ (1982)	Caternary curve	Incisal edges only
17	Richmond & Jones (1989)	Parabola	Middle curve of lower arch

Here, in this study by using the Ramanujan's formula for prediction of perimeter of ellipse we arrived at a result that (1) for every millimeter of inter-molar expansion 0.684mm increase in arch length is seen. (2) for every millimeter of incisor advancement 1.035mm increase in arch length is seen. (3) for every millimeter of incisor retraction, the arch length decreases by 0.661mm. (4) arch length decreases by 0.27mm for every 1mm reduction of inter-molar width.

Prediction of arch perimeter of an ellipse by Ramanujan's equation in this study can be effectively implemented with an acceptable accuracy of 1.281% error. This result is similar to the results of the study demonstrated by Chung and Wolfgramm[6] in which they correlated measured arch perimeter and calculated perimeter by equating in Ramanujan's formula for perimeter of ellipse of 30 diagnostic casts. The accuracy was with 1.2% error.

Numerous researchers conducted extensive studies for approximating various dental arch-forms to geometric shapes. Different amounts of arch perimeter change have been proposed for the same amount of expansion. Ricketts et al[16] suggested guidelines that each millimeter of canine expansion provides for a 1mm increase in arch perimeter, and that 1mm of molar expansion increased the perimeter by 0.25mm. Adkins et al[17] estimated the arch perimeter gained in patients treated with a hyrax expander was averagely 4.7mm while in molar expansion was 6.5mm. This indicated average 0.72mm perimeter increase for every 1mm of expansion.

Akkaya et al[18] showed that arch perimeter gain through the treatment could be predicted as 0.65 times the amount of the posterior expansion for rapid maxillary expansion and 0.60 times the amount of posterior expansion for slow maxillary expansion. Germane et al[10] developed a mathematical model using a spline function to compare various types of orthodontic expansions. They showed that increasing the midline arch length by incisor advancement was nearly 4 times as effective in increasing arch perimeter as was molar expansion. When the canine width and incisor positions were held constant, an initial 1mm increase in molar width produced approximately a 0.27mm increase in perimeter, the second millimeter produced an increase of 0.31mm, and the fifth millimeter of molar width increase was related to a perimeter increase of 0.41mm. When the incisor positions were fixed, each millimeter of canine expansion provided a 0.73mm increase in arch perimeter. When arch perimeter was increased by incisor advancement, it was nearly 4 times as effective in increasing arch perimeter as was molar expansion, depending on arch constriction.

Hnat et al[11] found that if the maxillary molar width is expanded by 6 mm (3mm per side) and the canine:molar expansion ratio is 1.25:1, then the arch perimeter alteration is +5.4mm. Correspondingly, the mandibular arch perimeter alteration is +5.6mm, when the 2 arches are in an Angle Class I occlusion.

Chung and Wolfgramm[6] stated that a 2mm expansion in inter-molar width resulted in 1.64mm expansion of inter-canine width. They predicted 1.47mm increase of arch perimeter for every 2mm expansion of molar-width, this result is similar to results in this study of 0.684mm increase in arch perimeter. They concluded that increase of 0.9mm arch perimeter is seen when 1mm expansion of inter-canine width occurs.

Study by Germane et al suggested that in cases of mild-moderate crowding with average arch-form, more than 5mm of molar expansion and 2mm of incisor advancement is required also 2.5mm of canine expansion is necessary to achieve the same increase in arch-perimeter.[10] Here, in crowding cases incisor advancement produced approximately 1.03mm of increase in arch-length.

Germane et al[10] in order to affect a 5mm increase in arch perimeter, approximately 5mm of combined molar-canine lateral expansion, versus a little more than 4mm of incisor proclination is necessary. In contrast, combined canine-incisor expansion of less than 2 mm can increase arch perimeter by 5mm.

The limitations of this study are that arch perimeter is not evaluated in canine and pre-molar region. Moreover second molars were not included in measurement of arch perimeter. This study was done only in maxillary arch without considering mandibular arch so further studies are required to evaluate arch-width changes. The complexity of the equation also is difficult to formulate and explain.

Whenever we face arch-length discrepancy (ALD) of less than 4mm, we rarely advice extraction depending upon the hard and soft tissue considerations. If ALD is about 5-9mm still it can be dealt with non-extraction taking into consideration the patient's chief complaint. ALD of more than 12mm have to be treated with extraction therapy. The current trend in orthodontics is also to expand he arches along with incisor proclination, but transverse expansion can give rise to complications of fenestration and dehiscence of roots from alveolar process of the bones. Also the stability outcome after the treatment should also be considered.

Conclusions:-

1. The perimeter of ellipse can be very well calculated by using Ramanujan's equation in cases of minor spacing and well-aligned arches.
2. The arch-length can be correlated to the perimeter with about 1.28% error.
3. Arch perimeter increase can be calculated effectively for molar expansion or incisor protrusion by using the equation for perimeter of ellipse.

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