



ISSN NO. 2320-5407

Journal homepage: <http://www.journalijar.com>
Journal DOI: [10.21474/IJAR01](https://doi.org/10.21474/IJAR01)

**INTERNATIONAL JOURNAL
OF ADVANCED RESEARCH**

RESEARCH ARTICLE

ORBIT DETERMINATION MODELING ANALYSIS BY GPS.

M.E. Awad¹, R. Ghoneim², Ahmed Adel Ahmed Abd-Elhamed³.

1. Professor of Space Dynamics, Cairo University, Faculty of Science, Egypt.
2. Associated Professor of Space Dynamics National Research Institute of astronomy and Geophysics (NRIAG), Egypt.
3. Master degree student in Cairo University, Faculty of Science, Egypt.

Manuscript Info

Manuscript History:

Received: 19 March 2016
 Final Accepted: 26 April 2016
 Published Online: May 2016

Key words:

orbit determination, GPS, RINEX.

***Corresponding Author**

Ahmed Adel Ahmed Abd-Elhamed

Abstract

The purpose of this paper was to analyze the modeling of a GPS satellite orbit, The Global Positioning System (GPS) is a satellite navigation system for determining position, velocity and time with high accuracy, using signals of the GPS constellation (RINEX files), with the aim of analyzing the performance of the orbit. One pursues to verify how differences of modeling can affect the final accuracy of orbit, To do that, the following effects were considered in the orbit propagation process: earth oblateness and Sun-Moon attraction. a RINEX(Receiver Independent EXchange) file with navigation data at the 8th of August 2015 was used as an input for an algorithm for determining the GPS satellite position and velocity , the algorithm was implemented using VB.net code with the ability of determining the position and velocity of any GPS satellite within the chosen orbit epoch of the used RINEX navigation file , in order to yield conclusive results about the position and velocity accuracy, a NGA(National Geospatial-Intelligence Agency) data file for precise GPS satellites positions and velocities at the chosen orbit epoch was used for the algorithm output verification. Numerical application will be given.

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Introduction:-

Orbit accuracy requirements can range from hundreds of meters or more for routine operations to a few centimeters for precise remote sensing. Among existing tracking systems, only GPS can meet the most stringent of these needs for the most dynamically unpredictable vehicles. The potential of GPS to provide accurate and autonomous satellite orbit determination was noted early in its development.

Gps orbit types:-

Below we have to confine ourselves to a short outline of the principles underlying the following orbit types:

- ❖ Precise Orbits produced by the Naval Surface Warfare Center together with the DMA, available upon request about 4-8 weeks after the observations.
- ❖ IGS orbits, produced by the International GPS Service for Geodynamics (IGS), available to the scientific world about 2 weeks after the observations.
- ❖ Broadcast orbits which are available in real time, their name indicates that they are transmitted by the satellites (Alfred Kleusberg, 1996).

Precise ephemerides:-

The precise ephemerides consist of satellite positions and velocities at equidistant epochs. Typical spacing of the data is 15 minutes. The format proposed by the NGS is widely used. Since 1985, NGS has been involved in the distribution of precise GPS orbital data.

Broadcast Orbit:-

The satellite positions estimated in the Kalman filter process are next represented in the form of Keplerian elements with additional perturbation parameters. The parameters refer to a given reference epoch, t_{0e} for the ephemeris and t_{0c} for the clock, and they are based on a four hours curve fit (ICD, 1993). Hence, the representation of the satellite trajectory is achieved through a sequence of different disturbed Keplerian orbits.

The first group of parameters is used to correct satellite time. The second group determines a Keplerian ellipse at the reference epoch, and the third group contains nine perturbation parameters.

Time parameters:-

t_{0e}	Reference time, ephemeris parameters [s]
t_{0c}	Reference time, clock parameters [s]
a_0, a_1, a_2	Polynomial coefficients for clock correction (bias [s], drift [s/s], drift-rate (ageing) [s/s ²])
IODC	Issue of Data, Clock, arbitrary identification number

Keplerian parameters:-

\sqrt{A}	Square root of the semi-major axis [m ^{1/2}]
e	eccentricity [dimensionless]
i_0	inclination angle at reference time [semicircles]
Ω_0	Longitude of ascending node at reference time [semicircles]
ω	Argument of perigee [semicircles]
M_0	Mean anomaly at reference time [semicircles]
IODE	Issue of Data, Ephemeris, arbitrary identification number

Perturbation parameters:-

Δn	Mean motion difference from computed value [semicircles/s]
$\dot{\Omega}$	Rate of change of right ascension [semicircles/s]
\dot{i}	Rate of change of inclination [semicircles/s]
C_{us}	Amplitude of the sine harmonic correction term to the argument of latitude [rad]
C_{uc}	Amplitude of the cosine harmonic correction term to the argument of latitude [rad]
C_{is}	Amplitude of the sine harmonic correction term to the angle of inclination [rad]
C_{ic}	Amplitude of the cosine harmonic correction term to the angle of inclination [rad]
C_{rs}	Amplitude of the sine harmonic correction term to the orbit radius [m]
C_{rc}	Amplitude of the cosine harmonic correction term to the orbit radius [m]

where

Δn is the secular drift in $d\omega/dt$ due to the second zonal harmonic (C_{20}); also it absorbs effects of the Sun's and Moon's gravitation and solar Radiation pressure over the interval of fit,

$\dot{\Omega}$ is the Secular drift in right ascension of the node due to the second zonal Harmonic; includes also effects of polar motion,

\dot{i} is the Rate of change of inclination, and

C_{us}, C_{uc} short periodic effects of C_{20} ; also include higher order effects and

C_{is}, C_{ic} short periodic effects of lunar gravitation (during the closest approach

C_{rs}, C_{rc} of the space vehicle to the Moon); also absorb further perturbations.

Fig. 1 explains the Keplerian and the perturbation parameters. Note that the element Ω_0 in the GPS message is not measured from the vernal equinox, Υ , but from the zero meridians, X_T . In essence, Ω_0 is not a right ascension angle but a longitude. In recent literature the parameter is therefore designated as longitude of ascending node (LAN).

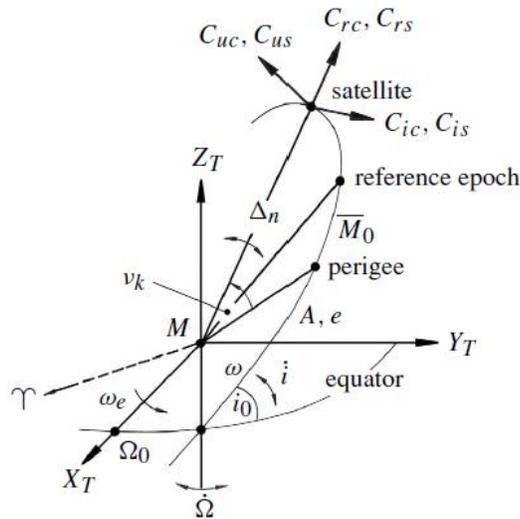


Figure 1:- Keplerian and disturbance parameters in the broadcast message.

Computation of Satellite Time and Satellite Coordinates:-

The GPS system time is a continuous time scale, and is defined by the weighted mean of the atomic clocks in the monitor stations and the satellites. Using

$$t = t_{SV} - \Delta t_{SV} \quad (1)$$

In which

$$\Delta t_{SV} = a_0 + a_1(t - t_{0c}) + a_2(t - t_{0c})^2; \quad (2)$$

The satellite coordinates X_k, Y_k, Z_k are computed for a given epoch, t , with respect to the Earth-fixed geocentric reference frame X_T, Y_T, Z_T . The time, t_k , elapsed since the reference epoch, t_{0e} , is

$$t_k = t - t_{0e}. \quad (3)$$

Then, we can use the following Computational Sequence to compute the GPS satellite position and velocity at given epoch

1. $A = (A^{1/2})^2$.
2. $n_0 = \sqrt{\frac{\mu}{A^3}}$
3. $t_k = t - t_{0e}$
4. $n = n_0 + \Delta n$
5. $M_k = M_0 + nt_k$
6. $\dot{M}_k = n$
7. $M_k = E_k - e \sin E_k$
8. Solve for E_k by iteration using NEWTON-RAPHSON method.
9. $E_k = M_k$
10. $\dot{E}_k = \dot{M}_k / (1 - e \cos E_k)$
11. $v_k = \tan^{-1} \left\{ \frac{\sin v_k}{\cos v_k} \right\} = \tan^{-1} \left\{ \frac{\sin E_k \sqrt{1-e^2}}{\cos E_k - e} \right\}$
12. $\dot{v}_k = \sin E_k \dot{E}_k (1 + e \cos v_k) / [(1 - \cos E_k e) \sin v_k]$
13. $\varphi_k = v_k + \omega$
14. $\dot{\varphi}_k = \dot{v}_k$
15. $\partial u_k = c_{us} \sin(2\varphi_k) + c_{uc} \cos(2\varphi_k)$
16. $\partial r_k = c_{rs} \sin(2\varphi_k) + c_{rc} \cos(2\varphi_k)$
17. $\partial i_k = c_{is} \sin(2\varphi_k) + c_{ic} \cos(2\varphi_k)$
18. $\partial \dot{u}_k = 2[c_{us} \cos(2\varphi_k) - c_{uc} \sin(2\varphi_k)] \dot{\varphi}_k$
19. $\partial \dot{r}_k = 2[c_{rs} \cos(2\varphi_k) - c_{rc} \sin(2\varphi_k)] \dot{\varphi}_k$

20. $\partial \dot{i}_k = 2[c_{is} \cos(2\varphi_k) - c_{ic} \sin(2\varphi_k)] \dot{\varphi}_k$
21. $u_k = \varphi_k + \partial u_k$
22. $r_k = A(1 - e \cos E_k) + \partial r_k$
23. $i_k = i_0 + (IDOT)t_k + \partial i_k$
24. $\dot{u}_k = \dot{\varphi}_k + \partial \dot{u}_k$
25. $\dot{r}_k = Ae \sin E_k \dot{E}_k + \partial \dot{r}_k$
26. $\dot{i}_k = IDOT + \partial \dot{i}_k$
27. $\dot{x}_k = r_k \cos u_k$
28. $\dot{y}_k = r_k \sin u_k$
29. $\dot{x}_k = \dot{r}_k \cos u_k - r_k \sin u_k \dot{u}_k$
30. $\dot{y}_k = \dot{r}_k \sin u_k + r_k \cos u_k \dot{u}_k$
31. $\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e t_{oe}$
32. $\dot{\Omega}_k = \dot{\Omega} - \dot{\Omega}_e$
33. $x_k = \dot{x}_k \cos \Omega_k - \dot{y}_k \cos i_k \sin \Omega_k$
34. $y_k = \dot{x}_k \sin \Omega_k + \dot{y}_k \cos i_k \cos \Omega_k$
35. $z_k = \dot{x}_k \sin i_k$
36. $V_x = \dot{x}_k \cos \Omega_k - \dot{y}_k \cos i_k \sin \Omega_k + \dot{y}_k \sin i_k \sin \Omega_k \dot{i}_k - y_k \dot{\Omega}_k$
37. $V_y = \dot{x}_k \sin \Omega_k + \dot{y}_k \cos i_k \cos \Omega_k - \dot{y}_k \sin i_k \cos \Omega_k \dot{i}_k + x_k \dot{\Omega}_k$
38. $V_z = \dot{y}_k \sin i_k + \dot{x}_k \cos i_k \dot{i}_k$ (Benjamin W. Remondi, 2004)

The previous algorithm for determination of GPS satellite position and velocity from the navigation data broadcasted by the GPS satellites through the navigation message (using RINEX file: 7odm2200.15n as an input) was implemented using Microsoft Visual Studio.NET and the results compared with Ephemeris data by NGA (using nga18566.eph file) were listed below.

Case 1:-

Initial data:-

Satellite number: 2

Orbit Epoch: 8 August 2015 00:00:00

Algorithm implementation results:-

	Algorithm	NGA		Algorithm	NGA
X(m)	15830503.92	15830503.028	V_x(m/s.)	1961.655558	1961.655800
Y(m)	-14537933.02	-14537933.5	V_y(m/s.)	-105.187985	-105.187890
Z(m)	15689703.93	15689704.25	V_z(m/s.)	-2172.175123	-2172.175211

	Algorithm	NGA
Clock Offset (Microseconds)	582.820270	582.819724

Conclusion:-

The differences between the algorithm implementation results and NGA precise ephemeris were:

In X = 0.892 m, in Y = 0.48 m, in Z = -0.32 m, in V_x = -0.000242 m/s.

In V_y = -0.000095 m/s and in V_z = 0.000088 m/s.

And the difference in the clock offset calculations was 0.000546 Microseconds

Case 2:-

Initial data:-

Satellite number: 21

Orbit Epoch: 8 August 2015 02:00:00

Algorithm implementation results:-

	Algorithm	NGA		Algorithm	NGA
X(m)	-9311806.31	-9311809.24	V_x(m/s.)	2030.014552	2030.014889
Y(m)	-13184783.77	-13184782.8	V_y(m/s.)	-1647.475574	-1647.475811
Z(m)	21791023.88	21791025.47	V_z(m/s.)	-167.974896	-167.974495

	Algorithm	NGA
Clock Offset (Microseconds)	-470.221042	-470.220520

Conclusion

The differences between the algorithm implementation results and NGA precise ephemeris were:

In X = 2.93 m, in Y = -0.97 m, in Z = -1.59 m, in V_x = -0.000337 m/s.

In V_y = 0.000237 m/s and in V_z = -0.000401 m/s.

And the difference in the clock offset calculations was -0.000522 Microseconds

Propagation of satellite State vector:-

To accurately predict position and velocity of a spacecraft through certain interval we have to take into account the perturbing forces acting on satellite during its motion around earth.

Perturbations due to Earth Gravitational Field:-

The Earth is not a perfect sphere with homogeneous mass distribution, and cannot be considered as a material point. Such irregularities disturb the orbit of an artificial satellite and the keplerian elements that describe the orbit do not stay constant. The perturbing function can be given by:

$$\tilde{R} = -\frac{3\mu J_2}{2r} \left(\frac{R_\oplus}{r}\right)^2 \left(\frac{\sin^2(i)}{2} - \frac{1}{3}\right) \tag{4}$$

Now apply the Lagrange VOP equations with a disturbing function, the secular change due to earth oblatness is

$$\dot{\Omega}_{sec} = -\frac{3nR_\oplus^2 J_2}{2P^2} \cos(i) \tag{5}$$

$$\dot{\omega}_{sec} = \frac{3nR_\oplus^2 J_2}{4P^2} \{4 - 5\sin^2(i)\} \tag{6}$$

$$\dot{M}_0 = \frac{-3nR_\oplus^2 J_2 \sqrt{1-e^2}}{4P^2} \{3\sin^2(i) - 2\} \tag{7}$$

Perturbations due to Third-Body Potential:-

These perturbations are due to Sun and Moon attraction force and they can be meaningful if the satellite is far from the Earth. As the orbital variations are of the same type, whatever is the Sun or the Moon the attractive body, they should be studied without distinguishing the third body. The luni-solar gravitational attraction mainly acts on Ω and ω, what causes precession of the orbit and the orbital plane. The secular change due to Third-Body Potential is:

$$\dot{\Omega} = \frac{3\mu_3 C}{4r_3^2 n^2 \sqrt{1-e^2} \sin^2(i)} \{5Ae^2 \sin(2\omega) + B[2 + 3e^2 - 5e^2 \cos(2\omega)]\} \tag{8}$$

$$\dot{\omega} = -\dot{\Omega} \cos(i) + \frac{3\mu_3 \sqrt{1-e^2}}{2r_3^2 n^2} \left\{ 5AB \sin(2\omega) + \frac{5}{2} (A^2 - B^2) \cos(2\omega) - 1 + \frac{3(A^2 + B^2)}{2} \right\} + \frac{15\mu_3 a (A \cos(\omega) + B \sin(\omega))}{4r_3^3 n^2 e r_3} \left[1 - \frac{5}{4} (A^2 + B^2) \right] \tag{9}$$

To accurately predict position and velocity of a spacecraft through certain interval taking into account the earth oblatness (J₂) and Luni-Solar attraction the following algorithm can be used.

Calculation of precession parameters z, θ, ζ using the following equations

$$z = 2306.'' 2181T + 1.'' 09468T^2 + 0.'' 018203T^3$$

$$\theta = 2004.'' 3109T - 0.'' 42665T^2 - 0.'' 041833T^3$$

$$\zeta = 2306.'' 2181T + 0.'' 30188T^2 + 0.'' 017998T^3$$

Where T is the measuring time in Julian centuries (36 525 days) counted from J2000.0.

Calculation of R_p from equation

$$R_p = R_3(-z)R_3(\theta)R_3(-\zeta)$$

$$= \begin{pmatrix} \cos(z) \cos(\theta) \cos(\zeta) - \sin(z) \sin(\zeta) & -\cos(z) \cos(\theta) \sin(\zeta) - \sin(z) \cos(\zeta) & -\cos(z) \sin(\theta) \\ \sin(z) \cos(\theta) \cos(\zeta) + \cos(z) \sin(\zeta) & -\sin(z) \cos(\theta) \sin(\zeta) + \cos(z) \cos(\zeta) & -\sin(z) \sin(\theta) \\ \sin(\theta) \cos(\zeta) & -\sin(\theta) \sin(\zeta) & \cos(\theta) \end{pmatrix}$$

Calculation of $\varepsilon, \Delta\Psi, \Delta\varepsilon$ and β from equations

$$\varepsilon = 84381.''448 - 46.''8150T - 0.''00059T^2 + 0.''001813T^3$$

$$\Delta\Psi = \sum_{i=1}^{106} (A_i + A_i'T) \sin\beta$$

$$\Delta\varepsilon = \sum_{i=1}^{106} (B_i + B_i'T) \cos\beta$$

$$\beta = N_{1i}l + N_{2i}l' + N_{3i}F + N_{4i}D + N_{5i}\Omega$$

Where l is the mean anomaly of the Moon, l' is the mean anomaly of the Sun, $F = L - \Omega$, D is the mean elongation of the Moon from the Sun, Ω is the mean longitude of the ascending node of the Moon, and L is the mean longitude of the Moon. The formulas of l, l', F, D , and Ω , are given in Sect. 11.2.8. The coefficient values of $N_{1i}, N_{2i}, N_{3i}, N_{4i}, N_{5i}, A_i, B_i, A_i', B_i', A_i'',$ and B_i'' can be found in, e.g., McCarthy (1996).

Calculation of R_N from equation.

$$R_N = R_1(-\varepsilon - \Delta\varepsilon)R_3(-\Delta\Psi)R_1(\varepsilon)$$

$$\approx \begin{pmatrix} 1 & -\Delta\Psi \cos\varepsilon & -\Delta\Psi \sin\varepsilon \\ \Delta\Psi \cos\varepsilon_t & 1 & -\Delta\varepsilon \\ \Delta\Psi \sin\varepsilon_t & \Delta\varepsilon & 1 \end{pmatrix}$$

Calculation of GAST, GMST, $GMST_0$ and α from equations.

$$GAST = GMST + \Delta\Psi \cos\varepsilon + 0.''00264 \sin\Omega + 0.''000063 \sin 2\Omega,$$

$$GMST = GMST_0 + \alpha UT1$$

$$GMST_0 = 6 \times 3600.''0 + 41 \times 60.''0 + 50.''5481 + 8640184.''812866T_0 + 0.''093104T_0^2 - 6.''2 \times 10^{-6}T_0^3$$

$$\alpha = 1.002737909350795 + 5.9006 \times 10^{-11}T_0 - 5.9 \times 10^{-15}T_0^2$$

Calculation of R_S from equation.

$$R_S = R_3(GAST)$$

Calculation of R_M from equation.

$$\approx \begin{pmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{pmatrix}$$

1. $r_{ECI} = [R_P]^T [R_N]^T [R_S]^T [R_M]^T r_{ECEF}$
2. $V_{ECI} = [R_P]^T [R_N]^T [R_S]^T \{ [R_M]^T V_{ECEF} + \omega_{earth} \times r_{ECEF} \}$
3. $h = \begin{bmatrix} I & J & K \\ r_I & r_J & r_K \\ v_I & v_J & v_K \end{bmatrix} = h_I I + h_J J + h_K K$
4. $n = -h_J I + h_I J$
5. $e = 1/\mu [(v^2 - \mu/r) - (r \cdot v) v]$.
6. $p = h^2/\mu$
7. $e = |e|$
8. $\cos i = h_K/h$
9. $\cos \Omega = n_I/n$
10. $\cos \omega = (n \cdot e)/ne$.
11. $\cos v_0 = (e \cdot r)/er$.
12. Check for the correct quadrant for the previous angles

13. Update the value of Ω changed by the secular effect of earth oblatness from equation (5):
14. Update the value of Ω changed by the secular effect of Luni-Solar attraction from equation (8).
15. Update the value of ω changed by the secular effect of earth oblatness from equation (6).
16. Update the value of ω changed by the secular effect of Luni-Solar attraction from equation (9).
17. Update the value of Mean Anomaly (M) :

$$M = M + n * (\text{Propagation Step Size})$$
18. Update the value of Mean Anomaly (M) changed by the secular effect of earth oblatness from equation (7).
19. $r = \frac{p}{1 + e \cos v}$
20. $r = r \cos v P + r \sin v Q$
21. $v = \sqrt{\frac{\mu}{p}} [-\sin v P + (e + \cos v) Q]$
22. $R_{11} = \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i$
23. $R_{12} = -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i$
24. $R_{13} = \sin \Omega \sin i$
25. $R_{21} = \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i$
26. $R_{22} = -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i$
27. $R_{23} = -\cos \Omega \sin i$
28. $R_{31} = \sin \omega \sin i$
29. $R_{32} = \cos \omega \sin i$
30. $R_{33} = \cos i$
31. $R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$
32. $r_{\text{ECI}} = R r^T$
33. $v_{\text{ECI}} = R v^T$
34. Update time: $t = t + (\text{Propagation Step Size})$.
35. Use R_p as calculated in step 2.
36. Use R_N as calculated in step 4.
37. Use R_S as calculated in step 6.
38. Use R_M as calculated in step 7.
39. $r_{\text{ECEF}} = [R_M][R_S][R_N][R_P] r_{\text{ECI}}$.
40. $v_{\text{ECEF}} = [R_M]\{[R_S][R_N][R_P] v_{\text{ECI}} - \omega_{\text{earth}} \times r_{\text{ECEF}}\}$.
41. Loop

The calculated state vector from the first algorithm was used as input data for propagation using the previous algorithm and the results were compared with reports generated by Satellite Tool Kit (AGI STK9) and the results depicted in the two figures below.

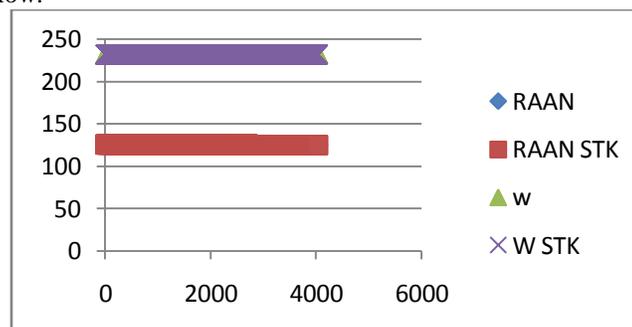


Figure 2:- Comparison between propagated values of Right ascension of ascending node and Argument of perigee.

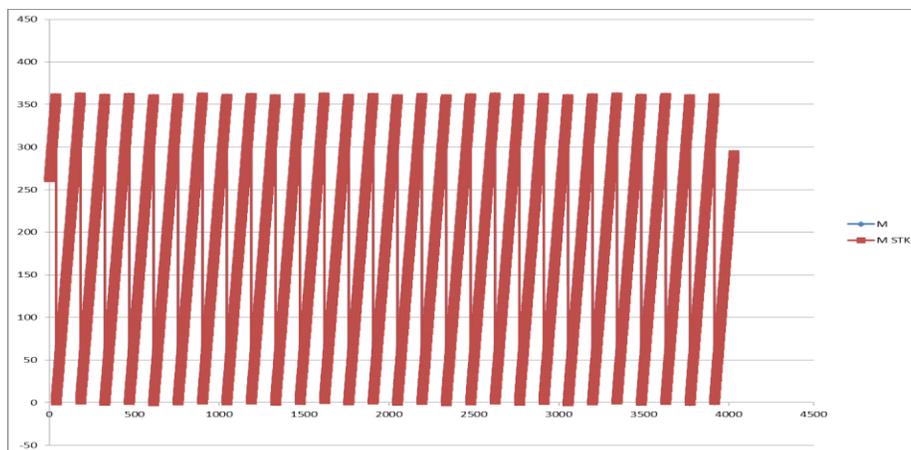


Figure 3:- Comparison between propagated values of Mean Anomaly.

Conclusions:-

This paper describes a satellite orbit determination concept, based on the broadcasted GPS navigation message, As can be seen in the results of the first algorithm, the achieved position accuracy lies in the order of 1 m for the position components and 0.0005 m/s for the velocity components.

The outlined GPS based satellite point positioning has been applied successfully in the next algorithm for motion propagation of the state vector and the compared results with STK were nearly identical as shown in the graphs.

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