ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

A METHOD FOR SOLVING FUZZY INVENTORY WITH SHORTAGE UNDER THE SPACE AND INVESTMENT CONSTRAINTS

[1]A.Faritha asma, [2]E.C.Henry Amirtharaj

1. Assistant professor, Department of Mathematics Government Arts college, Trichy-22. Tamilnadu, India.
2. Associate professor, Department of Mathematics, Bishop Heber college, Trichy-17.Tamilnadu, India.

Manuscript Info

Manuscript History:

Received: 17 November 2015
Final Accepted: 18 December 2015
Published Online: January 2016

Key words:
Inventory, space constraint, investment constraint, EOQ, Interval number, Fuzzy sets, triangular fuzzy number, Robust's ranking, Fuzzy optimization technique, Multi-objective Programming.

Corresponding Author

A.Faritha asma.

Abstract

This paper discusses an Economic Order Quantity (EOQ) model with shortage under the space, investment constraints, where the setup cost, the holding cost, price per unit, the shortage cost, demand, storage area and the investment amount are considered as triangular fuzzy numbers. The fuzzy parameters in the constraints are then transformed into crisp using Robust's ranking technique. The fuzzy parameters in the objective function are then transformed into corresponding interval numbers. Minimization of the interval objective function (obtained by using interval parameters) has been transformed into a classical multi-objective EOQ problem. The order relation that represents the decision maker's preference among the interval objective function has been defined by the right limit, left limit, and center which is the half –width of an interval. This concept is used to minimize the interval objective function. The problem has been solved by fuzzy programming technique. Finally, the proposed method is illustrated with a numerical example.

Introduction:-

In traditional mathematical problems, the parameters are always treated as deterministic in nature. However, in practical problem, uncertainty always exists. In order to deal with such uncertain situations fuzzy model is used [1],[15],in such cases, fuzzy set theory, introduced by Zadeh [17] is acceptable. There are several studies on fuzzy EOQ model. Lin et. al. [7] have developed a fuzzy model for production inventory problem. Katagiri and Ishii [5] have proposed an inventory problem with shortage cost as fuzzy quantity.

The parameters in any inventory model are normally variable uncertain, imprecise and adoptable to the optimum decision making process and the determination of optimum order quantity becomes a vague decision making process. The vagueness pertained in the above parameters analyze the inventory problem in a fuzzy environment.

On the basis of this idea the Robust's ranking method[9,11] has been adopted to transform the fuzzy constraints to a crisp one so that we can get the crisp constraints.

This paper discusses a fuzzy EOQ model with shortage together with the space and investment constraint. Demand, Holding cost, ordering cost, shortage cost, price per unit, storage space and investment are taken as triangular fuzzy numbers, and expression for fuzzy cost is established. For minimizing the cost function we transformed the fuzzy objective function into interval objective function. Now, this single objective function is then converted to multi-objective problem by defining left limit, right limit and center of the objective function. This multi- objective is then solved by fuzzy optimization technique. Linear membership function is considered here. This model is illustrated by a numerical example.

The article is organized as follows: In section 1 preliminary definitions of fuzzy set, interval number, triangular fuzzy number, α-cut of a fuzzy number, basic arithmetic optimization in interval, Robust's ranking technique and nearest interval approximation is briefly described. Section 2 contains model formulation. The fuzzy optimization technique is section 3. In section 4 the process is illustrated by a numerical example and in the last section the entire work is concluded.

Preliminaries:-

Definition 1:

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) A={(x, μ_A(x)) ; x$\in$ X.}, here μ_A:X$\rightarrow$ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of x$\in$ X in the fuzzy set A.

Definition 2:

Let $\Re$ be the set of all real numbers. An interval, may be expressed as

$$\bar{a} = [a_L, a_R] = \{x : a_L \le x \le a_R, a_L \in \Re, a_R \in \Re\} \quad \textbf{(1)}$$

where a_L and a_R are called the lower and upper limits of the interval $\bar{a}$, respectively.

If $a_L = a_R$ then $\bar{a} = [a_L, a_R]$ is reduced to a real number a, where a= a_L= a_R. alternatively an interval $\bar{a}$ can be expressed in mean-width or center-radius form as $\bar{a} = \langle m(\bar{a}), w(\bar{a}) \rangle$,where $m(\bar{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\bar{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\bar{a}$ .the set of all interval numbers in $\Re$ is denoted by I($\Re$).

Optimization in interval environment:-

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \bar{Z}(x) = \frac{\sum_{i=1}^{n} [a_{L_i}, a_{R_i}] \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} [b_{L_i}, b_{R_i}] \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(2)}$$

subject to x_j >0, j=1,2,…,n and x $\in$ S$\subset\Re$ where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}$, $0 < b_{L_i} < b_{R_i}$ and r_i ,q_j are positive numbers. Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\bar{Z}(x)$ can be written in the form $\bar{Z}(x) = [Z_L(x), Z_R(x)]$

where
$$Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i} \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(3)}$$

$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i} \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(4)}$$

The center of the objective function

$$Z_C(x) = \frac{1}{2}[Z_L(x) + Z_R(x)] \quad \textbf{(5)}$$

Thus the problem (2) is transformed in to

Minimize $\{Z_L(x), Z_R(x); x \in S\}$ **(6)**

subject to the non-negativity constraints of the problem, where Z_L, Z_R are defined by (4) and (5).

Definition 3: (**Triangular fuzzy number**)**:-**

For a triangular fuzzy number A(x), it can be represented by A(a,b,c:1) with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}; a \le x \le b \\ 1 \quad ; x = b \\ \frac{(c-x)}{(c-b)}; b \le x \le c \\ 0 \quad ; otherwise \end{cases}$$

Definition 4:(α-cut of a fuzzy number):-

The α-cut of a fuzzy number A(x) is defined as A(α)={x:μ(x)≥α, α∊[0,1]}

Robust's Ranking Technique:-

Given a convex fuzzy number $\tilde{a}$, the Robust's ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha{}^L, a_\alpha{}^U)d\alpha,$$

where $\left(a_\alpha{}^L, a_\alpha{}^U\right)$ is the α-level cut of the fuzzy number $\tilde{a}$.

In this paper we use this method of ranking for the fuzzy numbers in the space and investment constraints. The Robust's ranking index R($\tilde{a}$) gives the representative value of the fuzzy number $\tilde{a}$. Hence using this we defuzzify the space and the investment constraint into crisp one.

Robust's ranking technique [9,11] which satisfies compensation, linearity and additivity properties and provides results which are consistent with human intuition.

We apply Robust's ranking method[9,11] to defuzzify the fuzzy storage space and the fuzzy investment amount then we can get the crisp storage space constraint and the crisp investment constraint.

Nearest interval approximation:-

According to Gregorzewski [3] we determine the interval approximation of a fuzzy number as: Let $\tilde{A}$ =(a_1,a_2,a_3) be an arbitrary triangular fuzzy number with a α-cut [$A_L(\alpha)$,$A_R(\alpha)$].

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 \left[a_1 + (a_2 - a_1)\alpha\right]d\alpha = \frac{a_1 + a_2}{2}$$

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 \left[a_3 - (a_3 - a_2)\alpha\right]d\alpha = \frac{a_2 + a_3}{2} \quad \textbf{(7)}$$

Therefore, the interval number considering $\tilde{A}$ as triangular fuzzy number is $\left[\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}\right]$.

Model formulation:-

In this model, an inventory with shortage together with a space and investment constraints are taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item subject to the constraints by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations:-

C_1: Holding cost per unit time per unit quantity.

C_2: Shortage cost per unit time per unit quantity.
C_3: Setup cost per period
D: The total number of units produced per time period.
A-The space required by each unit(in sq.mt)
B-Maximum available ware house space (in sq.mt)
Q_1: The amount which goes into inventory
Q_2: The unfilled demand
Q: The lot size in each production run.

Assumption:-

(i) Demand is known and uniform.
(ii) Production or supply of commodity is instantaneous.
(iii) Shortages are allowed.
(iv) Lead time is zero

Let the amount of stock for the item be Q_1 at time t=0 in the interval $(0,t(=t_1+t_2))$, the inventory level gradually decrease to meet the demands. By this process the inventory level reaches zero level at time t_1 and then shortages are allowed to occur in the interval (t_1,t). The cycle repeats itself.(Fig. 1)

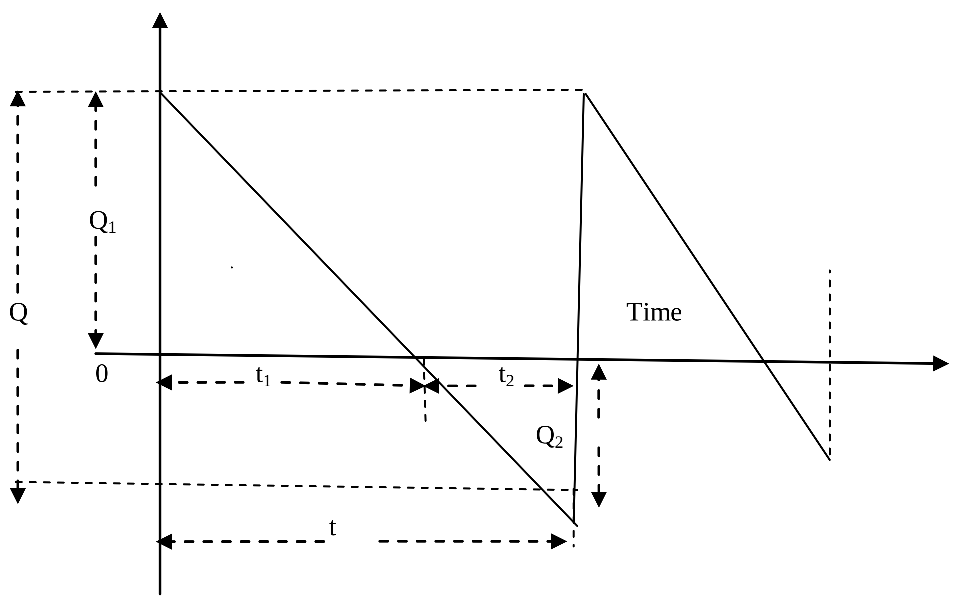

Fig:1 Inventory level for an item

The order level Q>0 which minimizes the average total cost C(Q) per unit time subject to the space and investment constraint is given by $\min C(Q) = \frac{1}{2}C_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_2\left(\frac{Q_2^2}{Q}\right) + C_3\left(\frac{D}{Q}\right)$

Subject to : $AQ_1 \leq B$ **(8)**
$C\,Q \leq F$

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e .of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demand and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demand $\tilde{D}=(D-\alpha,D,D+\beta)$ fuzzy holding cost $\tilde{C}_{1`}=(C_1-\alpha,C_1,C_1+\beta)$,fuzzy shortage cost $\tilde{C}_2=(C_2-\alpha,C_2,C_2+\beta)$, fuzzy ordering cost $\tilde{C}_3=(C_3-\alpha,C_3,C_3+\beta)$, fuzzy storage space $\tilde{B}=(B-\alpha,B,B+\beta)$,fuzzy price per unit $\tilde{C}=(C-\alpha,C,C+\beta)$,fuzzy investment $\tilde{F}=(F-\alpha,F,F+\beta)$.Replacing the real valued variables D,C_1,C_2 ,C_3,B,C & F by the triangular fuzzy variables $\tilde{D},\tilde{C}_1,\tilde{C}_2,\tilde{C}_3,\tilde{B},\tilde{C},\tilde{F}$ we get,

$$\min \tilde{C}(Q)=\frac{1}{2}\tilde{C}_1\left(\frac{Q_1^{\,2}}{Q}\right)+\frac{1}{2}\tilde{C}_2\left(\frac{Q_2^{\,2}}{Q}\right)+\tilde{C}_3\left(\frac{\tilde{D}}{Q}\right)$$

Subject to: $$AQ_1\le\tilde{B} \qquad \tilde{C}Q\le\tilde{F} \qquad \textbf{(9)}$$

Now applying Robust's ranking technique [9,11] to the space constraint and to the investment constraint we get

$$AQ_1\le R(\tilde{B})$$
$$R(\tilde{C})Q\le R(\tilde{F})$$

Where $R(\tilde{B}),R(\tilde{C})\,\&\,R(\tilde{F})$ are crisp values and can be calculated as

$$R(\tilde{B})=\int_0^1 0.5(B_\alpha{}^L,B_\alpha{}^U)d\alpha$$
$$R(\tilde{C})=\int_0^1 0.5(C_\alpha{}^L,C_\alpha{}^U)d\alpha$$
$$R(\tilde{F})=\int_0^1 0.5(F_\alpha{}^L,F_\alpha{}^U)d\alpha$$

Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [6] , the fuzzy numbers are transformed into interval numbers as

$\tilde{D}=(D-\alpha,D,D+\beta)$ =[D_L,D_R]

$\tilde{C}_{1`}=(C_1-\alpha,C_1,C_1+\beta)$ =[C_{1L},C_{1R}]

$\tilde{C}_{2`}=(C_2-\alpha,C_2,C_2+\beta)$ = [C_{2L},C_{2R}]

$\tilde{C}_3=(C_3-\alpha,C_3,C_3+\beta)$ =[C_{3L},C_{3R}].

Using the above expression (9) becomes

$$\tilde{C}(Q)=\left[f_{L,}\,f_R\right]$$

Where,

$$f_L=\frac{1}{2}C_{1L}\left(\frac{Q_1^{\,2}}{Q}\right)+\frac{1}{2}C_{2L}\left(\frac{Q_2^{\,2}}{Q}\right)+C_{3L}\left(\frac{D_L}{Q}\right)$$

Subject to : $$AQ_1\le R(\tilde{B}) \qquad R(\tilde{C})Q\le R(\tilde{F}) \qquad \textbf{(10)}$$

$$f_R = \frac{1}{2}C_{1R}\left(\frac{Q_1^{\ 2}}{Q}\right) + \frac{1}{2}C_{2R}\left(\frac{Q_2^{\ 2}}{Q}\right) + C_{3R}\left(\frac{D_R}{Q}\right)$$

Subject to : $AQ_1 \le R(\tilde{B})$
$R(\tilde{C})Q \le R(\tilde{F})$ **(11)**

The composition rules of intervals are used in these equations.
Hence the proposed model can be stated as
Minimize $\{ f_L(Q), f_R(Q)\}$,

Subject to the given constraints. **(12)**

Generally, the multi-objective optimization problem(12), in case of minimization problem, can be formulated in a conservative sense from (6) as
Minimize $\{ f_C(Q), f_R(Q)\}$,

Subject to : $AQ_1 \le R(\tilde{B})$
$R(\tilde{C})Q \le R(\tilde{F})$
$Q \ge 0$. **(13)**

Where $f_C = \frac{f_L + f_R}{2}$.

Here the interval valued problem (12) is represented as
Minimize $\{ f_L(Q), f_C(Q), f_R(Q)\}$,

Subject to : $AQ_1 \le R(\tilde{B})$
$R(\tilde{C})Q \le R(\tilde{F})$
$Q \ge 0$ **(14)**

The expression (14) gives a better approximation than those obtained from (12).

Fuzzy programming technique:-

To solve multi-objective minimization problem given by (14), we have used the following fuzzy programming technique.

For each of the objective functions $f_L(Q), f_C(Q), f_R(Q)$,subject to the space and investment constraint we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(Q), f_C(Q), f_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $f_k(Q)$, k=L,C,R. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique is given below.

Step 1: Solve the multi-objective cost function subject to the constraints as a single objective cost function subject to the constraints using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The fuzzy model of (14) can then be stated as, in terms of the aspiration levels for each objective, as follows: find Q satisfying $f_k \tilde{<} L_k, k = L, C, R$ subject to the space constraint and investment and non negativity conditions.

Step 4: Define fuzzy linear membership function $\left(\mu_{f_k}; k = L, C, R\right)$,for each objective function it is defined by

$$\mu_{f_k} = \begin{cases} 1 & ; f_k \le L_k \\ 1 - \dfrac{f_k - L_k}{d_k} & ; L_k \le f_k \le U_k \\ 0 & ; f_K \ge U_k \end{cases} \quad \textbf{(15)}$$

Step 5: After determining the linear membership function defined in(15) for each objective functions following the problem (14) can be formulated an equivalent crisp model as

$$\begin{aligned} &\text{Max } \alpha, \\ &\alpha \le \mu_{f_k}(x); k = L, C, R. \\ &\text{Subject to: } AQ_1 \le R(\tilde{B}) \\ &R(\tilde{C})Q \le R(\tilde{F}) \\ &\alpha \ge 0,\ Q \ge 0. \end{aligned} \quad \textbf{(16)}$$

Numerical example:-

In this section, the above mentioned algorithm is illustrated by a numerical example.

Here the parameters demand, ordering cost, holding cost and shortage cost are considered as triangular fuzzy numbers (TFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [3].Also the parameters storage area, investment amount are considered

Let C_1=5, C_2=25, C_3=100, D=5000, A=0.5 sq.mt, B=150sq.mt, C=6, F=1000.

Taking these as triangular fuzzy numbers we have,

$\tilde{C}_1$ =(3,5,7), $\tilde{C}_2$ =(21,25,31), $\tilde{C}_3$ =(85,103,109), $\tilde{D}$ =(4000,5000,6000), $\tilde{C}$ =(5.5,6,7.5), $\tilde{B}$ =(146,151,161),

$\tilde{F}$ =(500,1000,2000)

Using Robust's ranking technique,

$R(\tilde{B})$ =R(146,151,161)=152.25

$R(\tilde{F})$ =R(500,1000,2000)=1125

$R(\tilde{C})$ =R(5.5,6,7.5)=6.25

The fuzzy numbers $\tilde{D}, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$ are transformed into interval numbers as,

$\tilde{D}$ =[D_L,D_R]=[4500,5500]

$\tilde{C}_1$ =[C_{1L},C_{1R}]=[4,6]

$\tilde{C}_2$ = [C_{2L},C_{2R}]=[23,28]

$\tilde{C}_3$ =[C_{3L},C_{3R}]=[94,106]

Individual minimum and maximum of objective functions f_L, f_C, f_R are given below:

Table 1

Objective functions	Optimize f_L	Optimize f_C	Optimize f_R
f_L	f_L'=2656.667	f_L'' =2657.279	f_L''' =2658.59

f_C	$f_C{}'$ =3154.69	$f_C{}''$ =3154.007	$f_C{}'''$ =3154.39
f_R	$f_R{}'$ =3686.05	$f_R{}''$ =3684.07	$f_R{}'''$ =3683.595

Now we calculate

$L_L = \min\left(f_L{}', f_L{}'', f_L{}'''\right)$=2656.667 $U_L = \max\left(f_L{}', f_L{}'', f_L{}'''\right)$=2658.59

$L_C = \min\left(f_C{}', f_C{}'', f_C{}'''\right)$=3154.007 $U_C = \max\left(f_C{}', f_C{}'', f_C{}'''\right)$=3154.69

$L_R = \min\left(f_R{}', f_R{}'', f_R{}'''\right)$=3683.595 $U_R = \max\left(f_R{}', f_R{}'', f_R{}'''\right)$=3686.05

Using the equation (16), we formulate the following problem as:

Max α

$$\frac{1}{2}(4)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(23)\left(\frac{Q_2^2}{Q}\right)+94\left(\frac{4500}{Q}\right)+(1.923)\alpha \le 2658.59$$

$$\frac{1}{2}(5)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(25.5)\left(\frac{Q_2^2}{Q}\right)+100\left(\frac{5000}{Q}\right)+(0.683)\alpha \le 3154.69$$

$$\frac{1}{2}(6)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(28)\left(\frac{Q_2^2}{Q}\right)+106\left(\frac{5500}{Q}\right)+(2.46)\alpha \le 3686.05$$

$0.5*Q_1 \le 152.25$;

$6.25*Q \le 1125$;

$\alpha \ge 0$; $Q \ge 0$.

(17)

Results and Discussions:-

The solution obtained from (17) is given in table 2 and 3.

Table 2: optimum value of α

Maximum α
0.74769

Table 3:**optimum results**

f_L^*	f_C^*	f_R^*	Q^*	Q_1^*
2657.15	3154.01	3684.21	180.00	150.79

Table 4: comparison table

model	C_1	C_2	C_3	B	F	C	Q*	Q_1*	C*(Q)	α
Crisp	5	25	100	150	1000	6	166.66	138.88	3347.2	--

Crisp	5	20	100	150	800	5	160	128	3445.0	--
Crisp	3	20	100	150	800	6	133.33	115.94	3923.91	--
Crisp	5	25	75	125	800	6	133.33	111.11	3090.2	--
Fuzzy	[4,6]	[23,28]	[94,106]	152.25	1125	6.25	180	150.7	[2657.15,3684.21]	0.7476

Conclusion:-

In this paper, we have presented an inventory model with shortage together with the storage space and investment constraints, where carrying cost, shortage cost, ordering or setup cost, demand, price per unit, investment amount are assumed as triangular fuzzy numbers instead of crisp values so as to make the inventory model more realistic. At first, the fuzzy storage constraint and investment constraint are converted into crisp using Robust's ranking technique. The expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is approximated by interval number. After that the problem of minimizing the cost function subject to the constraints is transformed into a multi-objective inventory problem subject to the constraints, where the objective functions are left limit, right limit and the center of the interval function. Fuzzy optimization technique is then used to found out the optimal results. A numerical example illustrates the proposed method.

Reference:-

[1] Bellman, R.E., Zadeh , L.A.; Decision-making in a fuzzy environment.
Management science;(1970);17(4),141164.

[2] Chen, S.H., Wang, C.C., Chang, S.M.; Fuzzy economic production quantity model for items with imperfect quality. International Journal of Innovative Computing , Information and Control ;(2007);3(1),85-95.

[3] Grzegorzewski,P.; Nearest interval approximation of a fuzzy number Fuzzy Sets and Systems; (2002); 130,321-330.

[4] Ishibuchi, H.,Tanaka, H., Multi objective programming in optimization of the interval objective function. European Journal of Operational Research; (1990);48,219-225

[5] Katagiri,H.,Ishii,H.; Some inventory problems with fuzzy shortage cost. Fuzzy Sets and Systems; 2000;111(1),87-97.

[6] Li, L., Kabadi, S.N.,Nair, P.K.; Fuzzy models for single-period inventory problem. Fuzzy Sets and Systems;2002; 132(3), 273-289.

[7] Lin, D.C., Yao, J.S.; Fuzzy economic production for production inventory. Fuzzy Sets and Systems; 2000;111, 465-495.

[8] Mahapatra, G. S., Roy, T.K.; Fuzzy multi-objective mathematical programming on reliability optimization model. Applied Mathematics and Computation; 2006;174(1), 643-659.

[9] Nagarajan.R and Solairaju.A., "Computing improved Fuzzy Optimal Hungarian Assignment Problems with Fuzzy costs under Robust Ranking Techniques", International Journal of Computer Application,Vol.6,No.4(2010)

[10] Park, K.S.;Fuzzy set theoretic interpretation of economic order quantity. IEEE Transactions on systems, Man and Cybernetics, SMC;(1987);17,1082-1084.

[11]Shuganipoonam ,Abbas. S.H.,Gupta V.K., "Fuzzy Transportation Problem of Triangular Numbers with α-cut and Ranking Techniques", IOSR Journal of Engineering, Vol.2.No.5,2012,P.P:1162-1164

[12] Susovan Chakraborty, Madhumagal Pal, Prasun Kumar Nayak, Solution of interval-valued manufacturing inventory models with shortages, International J. of Engineering and Physical Sciences, 4(2) (2010) 96-101.

[13] Susovan Chakraborty, Madhumagal Pal, Prasun Kumar Nayak, Multi section technique to solve interval-valued purchasing inventory models without shortages, Journal of Information and Computing Sciences, 5(3), (2010) 173-182.

[14] Tang, S.; Interval number and fuzzy number linear programming. Fuzzy Sets and Systems; (1994); 66, 301-306.

[15] Wolfe, M.,A.; Interval mathematics, algebraic equations and optimization; Journal of Computational and Applied Mathematics; (2000); 124, 263-280.

[16]Yager.R.R., “A Procedure for Ordering Fuzzy Subsets of the Unit Interval”, Information Sciences, 24,(1981),143-161.

[17] Zadeh, L.A.; Fuzzy sets. Information and control;(1965); 8,338-352.

[18] Zimmermann, H.J.; Description and optimization of fuzzy system. International Journal of general system; (1976) ; 2(4),209-215.

[19] Zimmermann, H.J.; Fuzzy linear programming with several objective functions. . Fuzzy Sets and Systems; (1978); 1, 46-55.