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RESEARCH ARTICLE

Ranking the Manufacturing Units of Basic Metals in India

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Abstract

In this paper, Data Envelopment Analysis (DEA) model is applied to evaluate the relative technical efficiency of Manufacturing units of Basic metals all over India during the period 2010-2011.We have considered 22 states as Decision Making Units (DMUs) in all over India each with 7 inputs and 4 outputs. The Banker, Charnes, Cooper (BCC) model is Input – oriented which allows Variable Returns to Scale (VRS) and units are ranked based on peer counts. A super efficiency DEA model, namely Andersen Petersen (AP) DEA, is applied to break the tie of ranks.

1. INTRODUCTION

Data Envelopment Analysis is a linear programming based tool which evaluates the relative efficiency of decision making units, with multiple inputs and outputs. It identifies a subset of efficient "best-practice" DMUs. Decision making units refers to similar type of organizations that consume a variety of identical inputs to produce a variety of identical outputs. In this study, we consider 22 states of India as the DMUs.

1.1 The objectives of the present study are

- To study the basic features of Annual Survey of Industries (ASI) National Industrial Classification Data (2008) with reference to 22states of India.
- To study the Relative Efficiency of states with respect to the manufacturing of basic metals.
- Identification of Efficient and Inefficient states based on the Efficiency scores.
- To construct the Peer group for the inefficient states so that the inefficient states could compare their Input and Output and works towards attaining efficiency.
- Ranking of DMUs based on Peer count.
- Applying AP- Model to break ties in ranks.

In this paper we apply an equivalent model in which super-efficiency scores can be obtained using the standard BCC models.

1.2 Review of Literature

DEA measures the efficiency of each decision-making unit by comparison with other units in the sample. The scores of the effieicnt DMUs determine a surface called as frontier surface. Efficiency measures are then calculated relative to this surface. A unit on the efficient frontier is given a score of 1, while units that do not lie on that surface get a score is the interval (0,1) and are identified as inefficient. The piece-wise-linear convex hull approach to frontier estimation, proposed by Farrell (1957), was considered by only a few authors in the two decades following Farrell paper. The mathematical programming method did not receive wide attention until the publication of the paper by Charnes, Cooper and Rhodes (1978), in which the term Data envelopment analysis was first used.These authors proposed a model that had an input orientation and output orientation under assumption of Constant Returns to Scale (CRS) that followed from fractional programming model. In the CRS version, it is assumed that increase in the amount of inputs would lead to a proportional increase in the amount of outputs after this initial study by Charnes, Cooper and Rhodes another model which permits Variable Returns to Scale (VRS) introduced by BCC (1984). In the VRS version, the amount of outputs is deemed to increase more or less proportionally than the increase in the inputs. The CRS version is more restrictive than the VRS and usually produces fewer numbers of efficient units and also lower efficiency scores for all DMUs. This is due to the fact that the CRS is a special case of the VRS model.

An important extension has been the creation during the past decade of ‘super-efficiency’ models. Super-efficiency data envelopment analysis (DEA) model was originally proposed by Andersen and Petersen (1993) to provide a ranking system that would help them discriminate between frontier firms by DEA model as well as AP-models. These deleted domain models exclude the efficient DMU under evaluation from the reference set. Dula and Hickman (1997), Seiford and Zhu (1999) prove conditions under which various super-efficiency models are infeasible. Despite these drawbacks, due to the simplicity of this concept, many researchers have used this approach. Zhu (2001) studied super efficiency and DEA sensitivity analysis. Lovell (2003) proposed Equivalent standard DEA models to provide super-efficiency scores. Chen (2005) considered super-efficiency DEA model in presence of infeasibility. Banker and Chang (2006) discussed super efficiency procedure for outlier identification and not for ranking efficient units. Subbarayan and Prakash (2009) carried out a performance based rankings of a selected set of states in India by DEA. Prakash , Rajesh and Thilagam(2012) study on technical efficiency of state road transport undertaking in India using DEA and identified that out of 34 DMUs, 7 DMUs are efficient and recommended that all the inefficient DMUs should reduce their input according to the radial value and slack movement to get the maximum output.

2. BASIC DEA MODELS

In DEA efficiency of a DMU is defined as the ratio of its weighted sum of outputs to its weighted sum of inputs. The first standard DEA model as proposed by Charnes, Cooper, and Rhodes (1978), in ratio form is expressed as follows: Let there be k DMUs and each unit has n outputs and m inputs. Efficiency of observed DMU (E_k) is defined as follows

$$\max \quad E_k \quad \frac{\sum_{n=1}^{s} u_n \; y_{nk}}{\sum_{m=1}^{r} v_m \; x_{mk}}$$

$$Subject \;\; to$$

$$\frac{\sum_{n=1}^{s} u_n \; y_{nl}}{\sum_{m=1}^{r} v_m \; x_{ml}} \leq 1$$

$$u_n \, , v_m \; \geq 0$$

Where $n=1,...,s \quad m=1,...,r \quad l=1,...,k$

y_{nk} = n^th output of observed DMU, u_n = weight of that output

x_{mk} = m^th input of observed DMU, v_m = weight of that input

The above fractional programming problem can be converted in to a linear program by normalizing denominator such that the weighted sum of inputs equals unity. Therefore, the above fractional programming problem reduces to output maximization problem as

$$\max \; E_k = \sum_{n=1}^{s} u_n \; y_{nk}$$

$$Subject \quad to$$

$$\sum_{m=1}^{r} v_m \, x_{mk} = 1$$

$$\sum_{n=1}^{s} u_n \, y_{nl} - \sum_{m=1}^{r} v_m \, x_{ml} \;\; \leq \; 0$$

$$u_n . v_m \geq 0$$

The above problem when it is solved, gives relative efficiency scores E_k ($0 \leq E_k \leq 1$), along with input and output weights. In general, a DMU is considered to be efficient if it obtains a score of 1 and inefficient if it gets score less than 1. According to the theory of duality in Linear Programming Problem (LPP) every primal has it own dual so the dual LPP corresponding to above primal is

$$\min \quad \theta$$

$$Subject \quad to$$

$$\sum_{i=1}^{k} x_{mi} \;\; \lambda_i \;\; \leq \;\; \theta \, x_{mk} \quad , \qquad m = 1,....,r$$

$$\sum_{i=1}^{k} y_{ni} \;\; \lambda_i \;\; \geq \; y_{nk} \quad , \qquad n = 1,.....,s$$

$$\lambda_i \;\; \geq \;\; 0 \qquad\qquad\qquad i = 1,.....,k$$

where θ is the Efficiency score

λ_i is the weight of DMUs

When the above model is solved for each DMU in the set, it gives an efficiency score θ and DMU weights λ_i The quantity θ is the factor needed to reduce the input of observed DMU to a frontier formed by its peers, or convex combinations of them, which produce no less output than observed DMU. The DMU will be efficient if θ equals one. If θ is less than one then DMU will be inefficient. Then the composite unit provides targets for the inefficient unit and θ represents the maximum inputs that a DMU should be using to attain at least its current output. The above model is Charnes, Cooper and Rhodes (CCR) which allows constant returns to scale. The other models know as BCC and it allows variable returns to scale. Banker, Charnes and Cooper modified CCR model by introducing the constraint $\sum_{i=1}^{k} \lambda_i = 1$ called convexity constraint. Input oriented BCC model has been applied here.

2.1 Input - Oriented BCC Model

$$\begin{aligned}
&\min \quad \theta \\
&\textit{Subject to} \\
&\quad \sum_{i=1}^{k} x_{mi} \ \lambda_i \ \le \ \theta \, x_{mk} \\
&\quad \sum_{i=1}^{k} y_{ni} \ \lambda_i \ \ge \ y_{nk} \\
&\quad \sum_{i=1}^{k} \lambda_i \ = 1 \\
&\quad \lambda_i \ \ge \ 0
\end{aligned}$$

where y_{ni}= nth output of the i th observed DMU
x_{mi}=mth input of the i th observed DMU
k= Number of DMUs
with meaning of the parameters being similar to the CCR model derived earlier.

2.2 Super efficiency

The methodology of AP-DEA Model enables an extreme efficient unit k to achieve an efficiency score greater than one by removing the kth constraint in the primal formulation. The model is described as follows

$$\begin{aligned}
&\min \quad \theta \\
&\textit{Subject to} \\
&\quad \sum_{\substack{i=1 \\ i \ne k}}^{k} x_{mi} \ \lambda_i \ \le \ \theta \, x_{mk} \\
&\quad \sum_{\substack{i=1 \\ i \ne k}}^{k} y_{ni} \ \lambda_i \ \ge \ y_{nk} \\
&\quad \sum_{i=1}^{k} \lambda_i \ = 1 \\
&\quad \lambda_i \ \ge \ 0
\end{aligned}$$

A DMU is efficient if its efficiency score is equal or greater than one, while all inefficient DMUs have score similar to ordinary DEA model.

3. DATA AND EMPIRICAL ANALYSIS

In this Paper secondary data are collected from “Manufacture of basic Metals published by Annual Survey of Industries” during the period 2010-2011. Three- two- digit level industry outputs and inputs are derived using National Industrial Classification (NIC) 2008. The industries manufacturing basic metals, Two digit classification: 24, this industry is constituted by Manufacture of basic iron and steel (241). We have considered 22 states which contribute to Nation economy. Here each state is considered as DMU.Each DMU is characterized by the following inputs and outputs viz Number of factories (Number),Fixed capital (Rs. Lakhs),Gross value of plant and machinery (Rs.Lakhs),Fuel consumed (Rs.Lakhs),Material consumed (Rs.Lakhs),Number of persons engaged (Number), Wages,Salaries. (Rs. Lakhs) and Total output (Rs. Lakhs), Net value Added (Rs. Lakhs), Gross Capital Formation (Rs. Lakhs), Income (Rs. Lakhs)

Table 1: Descriptive Statistics

Variables	Mean	Standard deviation	Minimum	Maximum
No.of Factories	196.45	34.92	24	551
Fixed Capital	1169099.50	382850.32	1690	6844595
Gross value of Plant and Machinary	952442.45	291684.21	787	4396764
Fuel Consumed	161202.05	43824.65	1636	795602
Meterial consumed	1183578.05	273916.42	20273	4793096
Number wages engaged	870479.73	362383.12	702	6563286
Wages and Salaries	46945.86	15470.91	406	238905
Total Output	1023070.14	368814.83	1083	5367514
Net value added	277057.27	82717.08	1485	1400002
Gross Capital Formation	407749.50	143849.89	554	2381765
Income	221172.68	69925.15	252	1231245

An on average 196 factories are operated in 22 states deriving the average income 221173 lakhs. It is found that among the 22 states maximum output as 5367514 lakhs. The average fixed capital per state is found to be 1169099.50 lakhs. The data on the other variables can be explained similarly.

Table 2: Efficiency Scores and Peers

DMUs	θ	Peers weights	Peer count	Ranks
Andhra Pradesh	1.000			
Assam	1.000			
Bihar	1.000		1	3
Chandigarh	1.000		3	1
Chattisgarh	1.000			
Dadra & N Haveli	0.959	λ_3 =0.243, λ_7 =0.744, λ_{10} =0.012		
Daman & Diu	1.000		1	3
Delhi	1.000			
Gujarat	1.000			
Haryana	1.000		1	3
Jharkhand	1.000		1	3
Karnataka	1.000			
Kerala	0.605	λ_4 =0.932, λ_{19} =0.035, λ_{20}= 0.000, λ_{21} =0.033		
Maharashtra	1.000			
Meghalaya	1.000			
Odisha	1.000			
Puducherry	1.000		1	3
Punjab	0.750	λ_4 =0.221, λ_{19}= 0.776, λ_{21}= 0.003		
Rajasthan	1.000		2	2
Tamil nadu	1.000		2	2
Uttar pradesh	1.000		3	1
Uttarakhand	0.697	λ_4 =0.632, λ_{11}= 0.003 λ_{17} =0.285, λ_{20} =0.039, λ_{21}=0.042		

Table 3: Super Efficiency Score

DMUs	Score	Ranks
Andhra Pradesh	1.057	14
Assam	1.393	7
Bihar	1.235	11
Chandigarh	6.389	1
Daman & Diu	6.260	2
Delhi	3.386	3
Haryana	2.735	4
Karnataka	1.738	6
Maharashtra	1.109	13
Meghalaya	1.254	12
Puducherry	1.352	8
Rajasthan	1.312	9
Tamil nadu	2.213	5
Uttar pradesh	1.236	10

4. CONCLUSION

In this study observed that 18 states are efficient and 4 states are inefficient among the 22 states under BCC model applied here. Dadra & N Haveli, Kerala, Punjab and Uttarakhand states are relatively inefficient DMUs. The Kerala state efficiency score is 0.605 which is lowest among the all inefficient DMUs identified in this study. It indicates the above DMU is 60% efficient in converting all its inputs, but wasting of 40% input. To operate efficiently, the above DMU should reduce all its inputs with reference to its peers.Ranking procedure have been carried out based on Peer counts. It is noticed that tie occurs among all efficient DMUs in respect of peer count. So the ranking procedure becomes complex and to break up the tie super efficiency model has been applied and rankings have been carried obtained, with the highest super efficiency scores Chandigarh state stood rank 1 , Daman & Die stood rank 2, Delhi stood rank 3 and so on .

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