A Scientific Approach for Evaluating Learning Effectiveness with respect to Different Parameters Associated with a Training Program

Abstract

1

2 3 4

29

30

31 32

33 34 **Purpose** - The purpose of the study is to evaluate the effectiveness of a five-day Project Management training program by establishing a relationship between the first (reaction) and second (learning) levels of the Kirkpatrick model of Evaluation of Training Results.

Design/methodology/approach – The study was conducted with 21 participants who attended the training program, which consisted of six different topics delivered by six faculty members. Pre and Post examinations were conducted for all sessions, and learning indices were calculated for each participant with respect to all faculty members. Factors affecting learning were identified based on participant feedback, and three factors i.e. Faculty Feedback Rating (FFR), Topic Difficulty Factor (TDF), and Optimal Session Time Factor (OSTF) were selected for evaluation. The relationship between the learning index (dependent variable) and the identified factor (independent variables) was established using multivariate linear regression.

Findings – The study demonstrated the effectiveness of using multivariate linear regression to establish a mathematical relationship between the learning index (associated with the second level of Kirkpatrick's model) and the independent variables (FFR, TDF, and OSTF) associated with the first level of Kirkpatrick's model. It was found that FFR, TDF, and OSTF significantly affected relative learning with respect to each session delivered by faculty members.

Research limitations/implications – The current research was conducted on a single training programme on a particular subject. More studies conducted with similar approach on other types of programmes on topics of different subjects/duration can be helpful in establishing the validity of approach. In this study, some identified factors from first (reaction) level of Kirkpatrick model were not considered applicable such as course design, background and experience of participants and environment/facilities. Training programmes conducted in different settings with heterogenous group of participants can be useful in studying the impact of these factor.

Originality/value – The idea of quantifying learning effectiveness by relating it to factors identified from first (reaction) level of Kirkpatrick model is a unique and original approach adopted in this study. Moreover, this study contributes to the field by providing a methodological approach to evaluate training effectiveness by linking the reaction and learning levels of the Kirkpatrick model using mathematical and statistical tools.

Keywords Learning effectiveness; Kirkpatrick's model of Evaluation of Training Results; Learning index; Faculty feedback rating; Topic difficulty factor; Optimal session time factor; Multivariate linear regression.

Paper Type Research Paper

1. Introduction

35 Learning is a continuous process by which the behavior of a learner is expected to change by an addition to previous knowledge & 36 experience. Training programs are designed and delivered with the intention of enhancing the learning of a target group of learners. 37 Learning Effectiveness of a training program can be considered proportional to knowledge gained by participants from it. Though 38 Learning Effectiveness is a qualitative terminology yet it can be quantitatively evaluated by a term called Learning Index (LI). In any training program if the Learning Index calculated is of a higher value, then it can be safely assumed that this training program 39 40 delivers more learning effectiveness. If examinations are conducted before (Pre) and after (Post) the training and a participant obtains P_i^e and P_i^f marks respectively, and maximum marks for examination is P_i^{max} then mathematically, Learning Index of ith participant is defined as $LI_i^i = (P_i^f - P_i^e) / (P_i^{max} - P_i^e)$. It is a non-dimensional quantity which provides quantitative representation of learning 41 42 effectiveness. However, effective learning in a training program depends on many factors which are explained in subsequent 43 44 paragraphs.

45 Faculty/ Trainer Effectiveness & Efficiency: The Trainer can play a vital role in effective training delivery. Trainers can typically hold many jobs, such as instructional designer, technical trainer or needs analyst. The trainer's role is to help trainees change their 46 47 behavior through the learning process. The trainer's teaching skills & techniques along with his/her personal characteristics play a 48 very important role in making a training program successful. Besides delivering knowledge, trainers also encourage and motivate the 49 trainees towards learning and further formulates performance benchmarks for trainees. There are two categories of competency for a 50 trainer, first one is basic (pedagogical) and another is specific (skills, abilities, aptitude/attitude and attributes). In the training program considered in this research work, an expert committee selected six faculty members, from different institutes and different 51 52 areas of specialization, who delivered lectures on different topics of Project Management. Faculty Feedback Rating (FFR), received 53 from participants, indicates trainer's effectiveness. Thus, FFR can be considered as one of the independent variables for evaluating 54 the Learning Index.

55 Teaching Methodologies: Teaching Methodologies are different ways of knowledge transfer from faculty to training participants. 56 Selecting a suitable teaching methodology for a specific topic plays a vital role in achieving effective knowledge transfer leading to 57 learning effectiveness. Trainers may adopt following teaching methodology depending on the topic (whichever is the best method for specific topic) of training program, for effective learning. As per requirement of topic/content, teaching methodologies are 58 59 categorized into four groups 1) trainer-centered methods, 2) learner-centered methods, 3) content-focused methods and 4) interactive/ participative methods. Each group includes some teaching methods e.g., Lecture Method, Discussion Method, Programmed Instruction, Study Assignment Method, Tutorial Method, Seminar Method, Demonstration Method, Group Task, Brainstorming, 60 61 62 Role Plays, Case Study, Hands on Practice etc. As per requirement of this particular training program under study, lecture method 63 has been adopted by all trainers. Lecture method is a way of relaying factual information which includes principles, concepts, ideas 64 and all theoretical knowledge about a given topic. In a lecture the trainer tells, explains, describes, or relates whatever information 65 the trainees are required to learn through listening and understanding. It is therefore trainer-centered. The trainer is very active, doing

all the talking. Trainees on the other hand, are relatively inactive, doing all the listening. In this training program, since all trainers
 used the lecture method for all participants, hence, teaching methodology did not affect the relative learning effectiveness of the
 participant with respect to trainers.

69 Course Design (Suitable Curriculum/Syllabus/Content): Contents of all topics in training programs are designed by an expert 70 team as per requirement of training participants. The training content usually refers to what is to be taught, at which level and in what 71 amounts. Training is more likely to be effective when the training content is linked to their current job experience and task assigned 72 and hence, gives meaning to them. Training transfer may be at a maximum when trainees learn the relevant training content applied 73 to their actual work environment and are able to practice their new skills. Likeness of training content to the actual job creates a 74 positive attitude toward the training activities. Content validity influences trainees' reactions and performance self-efficacy. In this 75 training program, an expert committee designed a course curriculum with their experience. They incorporated previous feedback of 76 the participants received in similar courses. Training content of all lectures have been designed by the same expert committee based 77 on particular requirements. Therefore, it did not affect relative learning effectiveness with respect to different faculty for different 78 lecture sessions.

Optimal time for given content: Learning effectiveness for a particular lecture/session depends on optimal time allotted to the trainer for covering contents of the topic(s) planned in the session. If the trainer spends excessive time for unnecessary elaboration of a topic, then participants may get exhausted after some time, resulting in reduced effectiveness. On the other hand, spending lesser time than required ensures fast delivery but resulting in the possibility of missing out important concepts. So, for deciding on optimal timing of sessions, feedback of participants have been obtained in terms of Adequacy of time for a particular lecture session. Optimal time for a session is measured from participant's feedback in terms of Optimal Time Factor (OTF).

Background & experience of participants: Training has been designed for a homogeneous group of training participants having similar age and skill, experience & qualification. In most of the cases students/participants may assess their own knowledge, skills, and expertise before selecting appropriate training programs to gain maximum knowledge. In this study, this factor has already been considered at the time of selection of training participants. Therefore, this factor does not contribute to deviation in relative learning effectiveness.

90 Environment/Facilities: Training environment/facilities affects the process of knowledge, which involves both knowledge 91 transmission by faculty as well as knowledge receipt by participants. All known environmental factors and training facilities for the 92 training program include arrangement of infrastructure and training facilities (like area and layout of classroom including open space, 93 seating layout & arrangement, gap between two rows, display resolution, illumination of the screen of the display board, colour of the board-white/black, marker/chalk position of classroom, etc.) and environmental factors (like classroom temperature and 94 95 humidity, how effectively temperature is maintained by air-conditioning and/or heater/blower, illumination of classroom, 96 cleanliness, undesirable odour etc.). Since all sessions have been conducted using same infrastructure and facilities, therefore, this 97 factor has not affected relative learning effectiveness.

Difficulty Level of Topics: Topics having different levels of difficulty and delivered by different faculties result in different levels of learning effectiveness. If topic A is more difficult than topic B and is delivered by multiple faculty members with similar ratings, it is likely that topic A having higher level of difficulty may receive slower response from participants compared to topic B, resulting in lesser transfer of knowledge. Thus, this factor has been considered to affect relative learning effectiveness of participants.

In the preceding paragraphs, it has been observed and discussed that learning effectiveness may depend on many factors associated with training, but, in this study, we will be moving forward with the idea that Learning Index of participant primarily depend on three main factors out of seven identified factors namely, Faculty/Trainer Effectiveness & Efficiency (Faculty Feedback Rating), Optimal time for given content (Optimal Time Factor) and Difficulty level of topics.

108 **2.** Literature review

107

Many relevant literatures in the form of articles from national and international journals, relevant presentations from conferences and symposia and other available write up on the topic were scanned, to study the evaluation of participant training effectiveness for training programs. The effectiveness of learning in training programs were studied in detail, from available literature of research already undertaken, with the aim of incorporating international best practices in the current study.

Tomic, W. (1991), this article contends that studies into the effectiveness of teacher behavior should give more attention both to a systematic design of training programs as well as to the collection of implementation data concerning teacher behavior, before incorporating the training program into an experimental design.

116 Moody, D. L., Sindre, G. (2003) they concluded that there is no standard instrument for evaluating learning effectiveness. While 117 final examinations and end-of-semester course evaluation surveys can be used to do this, they are not designed for this purpose, and 118 there are inherent problems using them in this way. This study describes a survey instrument, called the Learning Effectiveness 119 Survey, which can be used to evaluate and improve the effectiveness of learning interventions. Learning effectiveness is evaluated in 120 the context of the learning goals of the course (short term learning), and in the context of the overall educational programme and 121 future working life (long term learning). The instrument also provides feedback on the intervention and how it could be improved. A 122 case study is described in which the instrument is used to evaluate the use of peer reviews as a learning activity in a requirements 123 analysis course. The instrument was found to have relatively high validity, but reliability was below acceptable levels. Some 124 interesting results were also found on the determinants of learning. In particular, attitude was found to have no effect on short term 125 learning, but was found to be the primary determinant of long-term learning. 126

127 Clayson, D.E. (2009) found out the relationship between student evaluations and learning. The view of the literature shows that 128 attempts to find such a nomological relationship has been complicated by practice, methodology, and interpretation. It is concluded 129 that the more objectively learning is measured, less likely it is to be related to the evaluations.

Bhanji et. al (2012) their study describe the Program evaluation remains a critical but underutilized step in medical education. 130 131 This study compared traditional and retrospective pre-post self-assessment methods to objective learning measures to assess which 132 correlated better to actual learning. Forty-seven medical students participated in a 4-hour pediatric resuscitation course. They 133 completed pre and post self-assessments on pediatric resuscitation and two distracter topics. Postcourse, students also retrospectively rated their understanding as it was precourse (the "retrospective pre" instrument). Changes in traditional and retrospective pre- to 134 postcourse self-assessment measures were compared to an objectives-based multiple-choice exam. The traditional pre to post self-135 136 assessment means showed an increase from 1.9 of 5 to 3.7 of 5 (p < 0.001); the retrospective pre to post scores also increased from 137 1.9 of 5 to 3.7 of 5 (p < 0.001). Although the group means were the same, individual participants demonstrated a response shift by 138 either increasing or decreasing their traditional pre to retrospective pre scores. Scores on the 22-item objective multiple-choice test 139 also increased, from a median score of 13.0 to 18.0 (p < 0.001). There was no correlation between the change in self-assessments and objective measures as demonstrated by a Spearman correlation of)0.02 and)0.13 for the traditional and retrospective pre-post 140 141 methods, respectively. Students reported fewer changes on the two distracters using the retrospective pre-post versus the traditional 142 method (11 vs. 29). Students were able to accurately identify, but not quantify, learning using either traditional or retrospective pre-143 post "self-assessment" measures. Retrospective pre-post self-assessment was more accurate in excluding perceived change in 144 understanding of subject matter that was not taught.

Zheng, L. Fluang, R., Yu, J. (2013) said about e-learning for in-service teachers in order to improve teacher's instructional skills.
16,264 primary and secondary school teachers participated and evaluated the effectiveness of e-training using Kirkpatrick's four
level model. The final passing rate of e-training was over 80%.

145

161

149 Chahal, A. (2013) her study said that Training and development enables employees to develop skills and competencies necessary 150 to enhance bottom-line results for their organization. It is a key ingredient in banking sector for organizational performance 151 improvement. It ensures that randomness is reduced and learning or behavioural change takes place in structured format. Training 152 and Development helps in increasing the job knowledge and skills of employees at each level and helps to expand the horizons of 153 human intellect and an overall personality of the employees. This paper analyses the status of various need analysis-based training 154 and development practices in Punjab National Bank and HDFC bank and explores the proposed link between the training and 155 employees' productivity by adopting development-based theory. The study makes use of statistical techniques such as percentage, 156 mean, standard deviation, standard error and coefficient of variation in analysing the data for finding the result. The result shown that 157 the Training in PNB & HDFC is average and there is scope for improvement in training. The perception of employees regarding the 158 Training and Development somewhat differs significantly on the basis of gender and designation. Consequently, the 159 recommendations support for the noteworthy of needs assessment of training which will bring a constructive worth in banking 160 sector.

Borate, N.S., Krishna, Dr. G. et. al. (2014), the study to evaluate the effectiveness of employee training programs in MNC with the help of Kirkpatrick's four level of evaluation model. Data analysis and questionnaire reliability test were done using SPSS 30 software and Cronbach's Alpha (alpha=0.7) respectively. Using a paired sample T-test found that the training program was more effective. Finally, the result of the hypothesis determines four levels that have a significant impact on the training program.

Suresh, K.C., Agrawal, M.R., Rao, R. KVS (2014), the study is made to know the effectiveness of the training program using five factors (such as objectives and needs, age, gender, training factors) and performance were taken to measure the training effectiveness of the employees in an automotive component manufacturing organization. The questionnaire was designed based on the literature review, by interviewing employees and feedback from the pilot study and administered through a field survey method. The data obtained was analyzed by adopting Percentage method, chi-Square test, ANOVA, Correlation method and T-test. Based on the major findings suitable recommendations have been suggested.

Borate, N.S., Gopalkrishna, Borate, S.L. (2014) have been investigating the case study. This study was conducted to evaluate the effectiveness of training programs in the quality department at MNC. The data survey questionnaire based on Kirkpatrick's model. The questionnaire includes indicators of attitude (Reaction), learning, behavior and results. This survey belongs to 330 employees of the quality department itself. The questionnaire validity was determined through university professors and reliability value (Cronbach's alpha) was found more than (alpha=0.7). A paired sample T-test has been concluded that the employees find the training program more effective. The mean of hypotheses was significantly higher than the theoretical mean giving the effectiveness of the overall training program.

Bagul, D.B. (2014) stated that the prime objective of research is to study the changes in skill, attitude, knowledge, behavior of employees after training. It also studies the effectiveness of Training on both individual and Organizational levels. The research exposed us to become familiar with the professional environment, working culture, behavior, oral communication & manners. Since, the training is a result-oriented process involving lot of time and expenditure, it is necessary that the training program should be designed with great care. For evaluating effectiveness of training a questionnaire has to be carefully prepared for participants in order to receive feedback.

Al-Mzary, M.M.M. et. al (2015) this study is to examine the attitudes of administrative leaders and administrative employees concerning the training courses provided, as well as the impact of training on employee job performance at Yarmouk University in Jordan. The study is carried at a Malaysian small and medium enterprise (SME). Findings indicated that training courses are related to the training needs of the employees to a medium degree, and that there are several conditions which determine selecting eligible employees for training. Results indicated also that there is relationship between effective training and employees' job performance. Based on the results of the study, several recommendations were provided.

Jonny (2016) said in their research paper to evaluate the effectiveness of Kirkpatrick model and Return on Investment of Training at PT XYZ. The result has shown several facts such as trainees' feedback score (410 out of 462) in term of reaction, the average final exam score (300 out of 366) in term of learning, the trainees' superiors' feedback score (300 out of 353) and ROI-Training was 58,88% above 15%. With these results, the company can conclude that the program is effective in nurturing its supervisory leaders.

196 Salah, M. R. A. (2016) The Success or failure of modern business organizations depends on the quality of their human resources. 197 Well trained and highly developed employees are considered as corner stone for such success. Hence the purpose of the study was to 198 investigate the relationship between training, development, training and development and employee's performance and productivity 199 in selected Jordanian Private Sector transportation companies located in the Southern region of Jordan. The study was based on set of 200 hypotheses that HOs: hypothesized no relationships between variables, while H1-H6 hypothesized the existence of relationships 201 between stated variables. A quantitative approach is used Relevant data was collected through structured questionnaire. Subjects for 202 the study consisted of 254 employees which constituted 60% of the total target population of 420 people. 254 structured 203 questionnaires were distributed to employees on job location, 212 questionnaires were returned and only 188 were suitable for 204 statistical analysis, SPSS version 16 has been used to for data analysis. Both descriptive and inferential statistics were used for data 205 analysis. The statistical tools were aligned with the objective of the research. For this purpose, frequency tables, percentages, means, 206 and standard deviations were computed and substantively interpreted. Inferential statistics like Pearson product moment correlation 207 coefficient (r) and linear regression were used to determine if there is a significant positive relationship existed between the 208 independent variables (training and development) and dependent variables (performance and productivity). The findings indicated 209 that training and development were positively correlated and claimed statistically significant relationship with employee performance and productivity. Analysis and interpretations were made at 0.05 level of significance. The study concluded that training and 210 211 development have important impact on employee performance and productivity. Therefore, it was recommended that effective 212 training programs and carefully set development plans should be provided to all employees to enable them to enhance their skills and 213 upgrade their knowledge. Finally, foreseeable future research can be conducted to cover other variables like (capabilities, 214 involvement so on) which might affect performance and productivity. 215

Saha, J. (2017), this study tried to give a general overview of training effectiveness measurement models with critical appreciation.

216

217

Rao, D.S., Vijaya, K.P. (2017), this study tried to focus on Kirkpatrick's four level model. With the help of this model, to find out the difference of opinion and relationship among variables of reaction (such as training management process, materials and course structure and satisfaction towards trainer). Data collected from 267 respondents out of 2,645 participants. Descriptive statistics were applied by SPSSv20 software for analysis. Result was found that needed to upgrade its machinery, equipment, quality of course material and competency of the faculty.

Angela, R.L. (2017), this article described the accurate interpretation of student and faculty rationing data and selection procedure of rationing questions for good evaluation for research.

225 Shivaraju et al. (2017) stated that Didactic lecture is one of the most widely accepted methods among teaching and learning 226 methodology. Because of time restriction and vast syllabus to be covered through lectures, feedback knowledge before and after the 227 lectures to assess the extent knowledge of learners gained provides the platform for feedback method to improve the lectures to make 228 it more receptive for students. Aims and objectives were to evaluate the knowledge of didactic lecture among students by giving pre-229 and post-test questionnaire-based evaluation technique. 2nd year MBBS students (4th and 5th term) after obtaining their consent for 230 voluntary participation, asked to take the pretest containing 10 questions on antiamoebic drugs, and the same 10 questions were 231 provided at the end of the lecture as a post-test questionnaire to assess the effectiveness of the teaching as well as the receptive power 232 of students and their pre- and post-lecture knowledge. Papers were valued on score basis and improvement, data recorded, interpreted, and analyzed. There was significant improvement in the recipient knowledge after post-lecture assessment when 233 234 compared to pretest. Out of 156 students, only 56 (35.90%) obtained scores between 5 and 8 and 100 (64.10%) were below 5. These 235 scores were improved in post-test by 78.21% (122) obtained scores between 5 and 8, while 21.79% (34) got scores more than 8 236 indicating the high recipient group reflecting good improvement in cognitive structure. Voluntary participation in such tests provides 237 feedback on teachers teaching effectiveness and adequacy of knowledge gained by learners. 238

Sanyal, S., Hisam, M. W. (2018) said that This paper studies the impact of Training and Development practices on the employee performance in the select Omani Public and Private sector banking organizations. The aim of the paper is to analyze the impact of training and development practices on employee performance. The study adopts descriptive research design and it imbibes both primary and secondary data. Convenience sampling method is applied for collecting the data through administering a structured questionnaire. The sample size for the study is 300. Statistical tools like Pearson Correlation Analysis, Regression Analysis and ANOVA were applied to test the proposed alternate hypothesis. The study concludes that Training and Development Practices have a positive influence on employee performance in the Omani Banking Industry.

Claude Müller et al. (2018) With flexible learning, students gain access and flexibility with regard to at least one of the following dimensions: time, place, pace, learning style, content, assessment or learning path. Zurich University of Applied Sciences (ZHAW) has launched a new flexible learning study format called FLEX, a blended learning design allowing students to be more flexible as to when and where they study. It reduces classroom learning time, replacing some of it with an e-learning environment for self-study that includes instructional videos. In a pilot phase, we conducted a semi-experimental study on the learning effectiveness of FLEX. Students' perceptions of the new study format FLEX were found to be positive. In addition, the final test results of students in the FLEX programme were similar to those of other students, despite classroom learning time was reduced by about half.

255 Choudhury, G. B., Sharma, V. (2019), Her study concentrates on reviewing various models for training effectiveness evaluation 256 and then identifies the most suitable model for research and development (R&D) organizations.

Alias, S.A., Mohd Ong, H.A. et. al (2019) stated that is to establish the relationship between training design factors (training content, training methods and training competency) and training effectiveness in the context of the public service in Malaysia. 215 public service employees participated in this study. The results from SEM-PLS analysis indicated that training design factors, namely training design, training method and trainer competency significantly influenced the effectiveness of training. Trainer competency made the highest contribution towards training effectiveness followed by training method and training content.

Heydari, M. R. et. al (2019) This study is designed to evaluate the effect of a workshop about new teaching and learning methods on the response, knowledge, and behavior of healthcare staff working a large city healthcare center. Kirkpatrick's program evaluation model showed that the workshop on new teaching and learning methods significantly improved the healthcare staff's satisfaction about the teaching environment of workshops, their knowledge about new teaching and learning methods and their behavior in performing workshops for teaching people. It is recommended that this teaching and learning methods workshop should be considered in educational programs for healthcare staff.

269 Bharthvajan R et. al (2019) The Current study is concerned with "a study on effectiveness of training and development in its 270 solutions (chenna)" in this study where the effectiveness of the employee are measured and studied. In this study the where 110 271 employees are taken out of 195 employees using random sampling method. Where the set of questions are given to the employees to 272 get their feedback about the changes after the training. The organization provides various kind of training to the employee. In this 273 study we are testing the effectiveness of the employee after the training and development. Where the 110 questions are distributed to 274 employees and answers are collected for findings, suggestions and conclusions. The answers given by the respondents are analysed 275 using chi-square and percentage method. After the finding where concluded that the objectives of this study and conclusions found in 276 this study meets same point. Where the objective of this study is get satisfied. In this study we found that there is significant 277 difference in employee than before. The training imparted meets the objectives like. 278

279 Kashif, A. R. et. al (2020) stated that Organizations are struggling hard for the success and attainment of competitiveness 280 utilizing skilled human resource. The particular problem discussed in this research is to determine the influence of training on 281 performance of employee and organization within the education sector of Rawalpindi and Islamabad. For getting the primary data 282 about the concepts of people, a survey through questionnaire comprising of 15 questions was carried out. The questionnaires were 283 sent to 300 people, in Rawalpindi/ Islamabad. To depict a good representative of the study, the sample size was chosen randomly. 284 The replies were collected on paper. The independent variables of the study were three in numbers (On job training, Training design, 285 Delivery Style) and Organizational performance is a dependent variable which is being affected by these independent variables by 286 mediation of employee's performance. The data has been tested on SPSS. To find the importance of these variables and to evaluate 287 the results Cronbach's Alpha, descriptive statistics, correlation, regression, and ANOVA were used. The results show that the 288 independent variables training design, delivery style and on the job, training has positive and significant relationship with the 289 dependent variable organizational performance by mediating variable employee's performance and clearly depicts the strong 290 variability among variables. 291

292 Munna, A. S., Kalam, M. A. (2021) their study about that the enhancement of teaching effectiveness. Teaching and learning 293 process can be defined as a transformation process of knowledge from teachers to students. It is referred as the combination of 294 various elements within the process where an educator identifies and establish the learning objectives and develop teaching resources 295 and implement the teaching and learning strategy. On the other hand, learning is a cardinal factor that a teacher must consider while 296 teaching students. The paper evaluated various academic journals, pedagogy, and inclusive practices to assess the teaching 297 effectiveness within the higher education setting. The objective of the research is to assess the teaching effectiveness in a higher 298 education setting. The research used experimental research methods (primarily reflection) using literary forms to analyse the theory 299 with the reinforcement of the practice from the university experiences. The research findings suggest that providing positive and 300 adequate formative and developmental feedback, introduction of role-play has a profound positive impact on the students' confidence 301 and self-esteem. It was also revealed that, active learning environment promotes inclusivity and improve the faculty and student 302 academic performances. The research findings will enable the educators to help create and implement an inclusive teaching and 303 learning environment to improve the learner's expectation and academic performance.

304 Mehale, K.D., Govender, C.M., & Mabaso, C.M. (2021) stated that Employee performance is a vital aspect within organisations 305 in South Africa (SA). It is argued that poor performance can be addressed through training and development. Performances should be 306 evaluated before and after training interventions to ensure that training was beneficial to the employees. The study intended to 307 establish whether training evaluation conducted after training in the SA financial sector measures employee performance 308 improvement. Most businesses invest in training and development interventions anticipating that employees will use what they have 309 learned to improve their performance. There is limited recent empirical research on SA financial sector training evaluation tools, 310 especially those that indicate employee performance improvement after training. The findings of this study indicate the following: 311 SA financial organisations frequently use levels 1-3 (satisfaction; learning; application) of the Kirkpatrick- Phillips training 312 evaluation tool; continuous employee performance improvement needs to be assessed more regularly, especially after training; and 313 levels 4-5 (results; ROI) of the Kirkpatrick-Phillips evaluation model are seldom measured due to a lack of skills, motivation, and 314 resources. There are significant implications for Human Resource Development (HRD) professionals and managers within the SA 315 financial sector. Since there is a positive significant association with training evaluation and employee performance, relevant 316 stakeholders must be aware that the purpose of training must be to improve and measure employee performance. This paper 317 contributes theoretically to HRD management practices, training evaluations, and performance improvement. The practical 318 contribution is the proposed Training Evaluation Framework for Performance Improvement for stakeholders to use to ensure that 319 HRD evaluations measures performance improvement.

320 Theobald, M. (2021) The present meta-analysis tested the effects of extended self-regulated learning training programs on 321 academic performance, self-regulated learning strategies, and motivation of university students. The literature search revealed 49 322 studies (5,786 participants) that met the inclusion criteria. A three-level meta-analysis based on 251 effect sizes revealed an overall 323 effect size of g = 0.38. The largest effect sizes were obtained for metacognitive strategies (g = 0.40) and resource management 324 strategies (g = 0.39) followed by academic performance (g = 0.37), motivational outcomes (g = 0.35), and cognitive strategies (g = 0.37) 325 0.32). Training effects varied for specific self-regulated learning strategies and ranged between 0.23 (rehearsal) and 0.61 (attention 326 and concentration). Moderator analyses revealed differential training effects depending on course design characteristics: Feedback 327 predicted larger training effects for metacognitive and resource management strategies as well as motivation. Cooperative learning

arrangements predicted larger training effects for cognitive and metacognitive strategies. The provision of learning protocols predicted larger training effects for resource management strategies. Moreover, training programs based on a metacognitive theoretical background reported higher effects sizes for academic achievement compared to training programs based on cognitive theories. Further, training programs that targeted older students and students with lower prior academic achievement showed larger effect sizes for resource management strategies. To conclude, self-regulated learning training programs enhanced academic performance, self-regulated learning strategies, and motivation of university students.

- 334
- 335 336

348

353 354

355

356

365

3. PROPOSED METHODOLOGY

337 In this research study, for establishing the relationship between learning index and three identified parameters affecting 338 learning effectiveness, participant's feedback has been taken and 'Learning Index' with respect to each session have been calculated. Participants' feedback form was designed on Likert scale and distributed to all participants who attended the 339 course on 'Basic Project Management'. Pre & Post evaluation of all participants have been carried out with respect to all 340 sessions of faculty to evaluate respective Learning Indices. The research concluded with the multivariate regression analysis 341 between parameters of learning and reaction levels of the well accepted Kirkpatrick model for evaluation of effectiveness 342 343 training programs, parameters. Learning index was considered to represent learning while reaction was captured by participant's feedback which provided information to evaluate Topic Difficulty Factor, Optimal Session Time Factor and 344 345 Faculty Feedback Rating.

346 Details of Tools and their Purpose:

347 In this study, MS Excel has been used for solving the mathematical equations.

349 Data Collection:

Feedback data has been collected from course participants who had attended training sessions. Data of Pre and Post Evaluation marks has been used to calculate Learning Index.

- 352 There are four sections; each section represents its data and calculation process that is given below:
 - Learning Index (LI) using Pre and Post Evaluation marks
 - Faculty Feedback Rating (FFR)
 - Optimal Session Time Factor (OSTF)
 - Topic Difficulty Factor (TDF)

1. Learning Index (LI): In this section, Learning Index has been calculated by Pre-evaluation test and Post evaluation test of training participants. Learning indices have been calculated with respect to each faculty (Questions selected from the content of respective faculty in Pre as well as Post evaluation). Only non-negative real values of learning indices selected with respect to faculties have been considered and other indeterminate as well as negative values have been considered as zeros. Pre and Post evaluation was conducted with 90 questions (number of questions being same in each section) from 6 faculties (delivered in 9 sessions). The questions from different sessions delivered by the same faculty have been clubbed. Number of questions with respect to faculties are F1 = 23, F2 = 15, F3 = 10, F4 = 15, F5 = 10 and F6 = 17.

364 The *Learning Index* of the evaluation of the effectiveness for the faculty's sessions as given below:

$$LI_{i}^{i} = \frac{P_{i}^{j} - P_{i}^{e}}{P_{i}^{max} - P_{i}^{e}}$$
..... (II) Where, $0 \le LI_{i}^{i} \le 1$

Where, Ll_j^i = the learning index of student j with respect to sessions of faculty i (j=1 to 21, i=1 to 6), P_i^f = Post evaluation marks for questions from sessions of faculty i, P_i^e = Pre evaluation marks for questions from sessions of faculty i, P_i^{max} = Maximum marks in the questions from the sessions of faculty i.

While calculating the learning indices, two special cases have been observed where the values of learning indices were obtained either negative or indeterminate. These two types of values are not considered for calculating Overall Learning Index with respect to sessions delivered by a particular faculty. Overall learning index LI_{F_i} with respect to sessions delivered by particular faculty will be calculated as given below:

374

(where j=1 to 21, i=1 to 6), n = Total no. of valid learning indices of students (excluding negative and indeterminate values) $n \le 21$.

377

378 Calculation process:379

$$LI_{F_1} = \frac{(LI_1^1 + LI_2^1 + LI_3^1 + \dots + LI_{21}^1)}{21}$$

$$281$$

$$LI_{F_1} = \frac{\begin{pmatrix} 0.33 + 0.08 + 0.4 + 0.33 + 0.25 + 0.43 + 0.53 + 0.69 + 0.67 + 0.73 + 0.54 + 0.47 + 0.6 + 0.63 \\ +0.44 + 0.45 + 0.56 + 0.57 + 0.69 + 0.67 + 0.08 \\ 21 \end{pmatrix}}{21}$$

$$LI_{F_1} = 0.48$$

384

Following table represents Overall Learning indices of the sessions delivered by six faculties

386 TABLE I. LEARNING INDEX OF EACH FACULTY W. R. T. EACH PARTICIPANT AND THE OVERALL LEARNING INDEX OF EACH FACULTY

	Learning Index								
No. of Participants	F1	F2	F3	F4	F5	F6			
1	0.33	0.71	0.71	0.56	0.1	0.41			
2	0.08	0.2	0.6	0.33	0.67	0.4			
3	0.4	0.43	0.83	0.33	0.13	0.47			
4	0.33	0.67	0	0.67	0.71	0.64			
5	0.25	1	0.25	0.18	0.17	0.29			
6	0.43	0.5	0.57	0.38	0.5	0.55			
7	0.53	0.4	0.5	0.56	0.6	0.36			
8	0.69	0.82	0.33	0.42	0.5	0.65			
9	0.67	0.43	0.5	0.67	0.17	0.15			
10	0.73	0.67	0.25	0.44	0.4	0.59			
11	0.54	0.6	0.4	0.11	0.56	0.53			
12	0.47	0.5	0.4	0.58	0.75	0.4			
13	0.6	0.6	0.75	0.4	-	0.18			
14	0.63	0.6	0.6	0.67	0.75	0.5			
15	0.44	0.67	-	0.5	0.78	0.71			
16	0.45	0.54	0.67	0.6	1	0.5			
17	0.56	0.57	0.5	0.6	0.57	0.5			
18	0.57	0.25	0.6	0.56	0.4	0.4			
19	0.69	0.57	0.67	0.29	0.63	0.63			
20	0.67	0.44	0.5	0.45	0.4	0.41			
21	0.08	0.82	0.75	0.5	0.7	0.53			
Average	0.48	0.57	0.52	0.47	0.52	0.47			

387

388 2. Faculty Feedback Rating (FFR): Faculty members play a key role in the development and enhancement of the 389 quality of learning experience. Participant's feedback is an effective tool for faculty evaluation, resulting in faculty 390 development as well as providing scope for enhancing the effectiveness in future programs. A feedback form for faculty 391 evaluation was developed and validated through peer review/brainstorming. A customized feedback form with specific questions is distributed in the beginning of the training program for evaluating the extent to which faculty of different 392 393 subjects/topic have been successful in reaching out to the advanced as well as the slow learners in the classroom. It was 394 instructed to the participants in the beginning for providing faculty feedback (reaction) immediately after each session. It was ensured by Course Coordinator. 395

Feedback form concentrated on the parameters related to training delivery such as coverage of topic by the faculty completely as per Lesson plan [A1], Methodology of session delivery [A2], Interaction between faculty and participants during session [A3], Relevance of contents as per topic [A4], Solution to queries of participants [A5], and Adequacy of allocated time/duration for topic [A6]. The primary purpose of such feedback is to help training team decide whether faculty can be used in future training program for that topic or not, or whether faculty can be used after feedback to faculty and subsequent improvement. For this study, faculty feedback was also used to measure training effectiveness.

402 For first five parameters i.e. A1, A2, A3, A4, A5 a well-defined six-point scale (6: Excellent, 5: Very Good, 4: Good, 3: 403 Satisfactory, 2: Poor, 1: Unsatisfactory) was designated and for sixth parameter i.e A6, a two point (1: Yes/Adequate & 0: No/ 404 Not Adequate) was designated. These parameters are essential for evaluating overall rating for each faculty and lecture duration. Overall feedback has been calculated as the function of rating giving by all participants with respect to above 405 parameters. For calculating population rating, 80/20 criteria was applied instead of averaging the rating of all participants. 406 407 Criteria is based on the assumption that 20% of outlying responses are random & biased and cannot be relied upon. Remaining 408 80% selected from the responses with higher frequency are considered as unbiased & reliable. The method has been termed 409 "Representative Response Rating (RRR) and the process followed is discussed in detail below.

Faculties of training program were rated by 21 training participants in respect to above six parameters with defined rating scales. Most of the ratings received were either **Very Good** or **Excellent** level. Duration of session was found adequate in the opinion of most of the training participants.

- Table II, represent the weighted average of population with respect to all associated parameters. Though a total of 21 students contributed their opinion in terms of rating, opinion/rating of 80 % of 21 i.e., 16.8 were considered for calculating weighted
- 415 controlled their opinion in terms of rating, opinion rating of 80 % of 21 i.e., 10.8 were c 416 average population rating. The method has been briefly described below.
- 410

421

422

423

- The method has been coined as "Representative Response Rating (RRR). Step wise calculation procedure of the same is as follows [Kanango, J. et. al (2023)]:
 - a. Arrange responses in different Parameter Rating in descending order of frequency of responses.
 - b. Calculate the sum of respective frequency of occurrences of all responses.
 - c. Calculate 80% of the above sum of the frequencies. The categories which add up to this 80% of the total sum are the selected categories.
- 424 d. Add till cumulative frequency of responses added reaches at least 80% of the total sum of frequencies calculated in
 425 (b) above. This is the desired cumulative value.
- 426 e. While desired cumulative value is reached, if the last frequency considered occurs multiple times in the data collected, then all occurrences of that frequency are included in the calculation of RRR.
- f. The remaining responses are not considered in calculation of RRR.
- 429
- 430TABLE II.
FACULTY 1CALCULATION PROCESS OF WEIGHTED AVERAGE OF PARAMETER RATING AND ASSOCIATED QUANTIFICATION (AQ) FOR SESSION OF

AQ	A1	AQ*A1	AQ	A2	AQ*A2	AQ	A3	AQ*A3	AQ	A4	AQ*A4	AQ	Α	AQ*A5
													5	
6	12	72	6	16	96	6	13	78	6	13	78	6	12	72
5	9	45	5	5	25	5	8	40	5	8	40	5	9	45
4	0	0	4	0	0	4	0	0	4	0	0	4	0	0
3	0	0	3	0	0	3	0	0	3	0	0	3	0	0
2	0	0	2	0	0	2	0	0	2	0	0	2	0	0
1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
Weigh	nted	$R_{F1}^{A1} = (12*6)$	Weig	hted	$R_{F1}^{A2} = (16*6)$	Weig	hted	$R_{F1}^{A3} = (13*6)$	Weig	hted	$R_{F1}^{A4} =$	Weight	ted	R_{F1}^{A5} =
Avera	ige	+ 9*5)/	age		+ 5*5)/	age		+ 8*5)/	age		(13*6 +	age		(12*6 +
		(12+9)	Avera	ige	(16+5)	Average (13		(13+8)	Average		8*5)/(13+8)	Averag	ge	9*5)/(12+9)
		= 5.57			= 5.76	= 5.62				= 5.62			= 5.57	

432

436

In table II, Weighted average has been calculated using six-point rating scale for each parameter (A1, A2, A3, A4, and A5).
Opinion or ratings have been contributed by 21 course participants for this research. Faculty Feedback Rating (FFR) for each faculty has been calculated as given below:

$$R_{F_i} = \frac{\left(\sum_{j=1}^{n} R_{F_i}^{A_j}\right)}{total no of A_i} \dots \dots \dots (I)$$

437 Where, R_{Fi} = Overall Faculty Rating for ith Faculty, $R_{F_i}^{A_j}$ =the rating of faculty Fi with respect to parameter Aj where i=1 to 6 438 and j=1 to 5.

439 Calculation process:

$$R_{F_1} = \frac{\left(R_{F_1}^{A_1} + R_{F_1}^{A_2} + R_{F_1}^{A_3} + R_{F_1}^{A_4} + R_{F_1}^{A_5}\right)}{5}$$

$$R_{F_1} = \frac{(5.57 + 5.76 + 5.62 + 5.62 + 5.57)}{5}$$

 $R_{F_1} = \frac{(28.14)}{5} = 5.63$

. .

440 441

- 442 443
- 444

445 TABLE III.

COMPARISON BETWEEN MEAN, MEDIAN AND 80/20 CRITERIA FOR FACULTY FEEDBACK RATING FOR SESSION OF FACULTY 1

Statistical Methods	[A1]	[A2]	[A3]	[A4]	[A5]	$\mathbf{R}_{\mathrm{F1}} = (\sum \mathbf{R}_{\mathrm{F1}}^{\mathrm{Aj}}) / \mathbf{A}_{\mathrm{j}}$
80% > (Opinion Criteria)	5.57	5.76	5.62	5.62	5.57	5.63
Mean	5.57	5.76	5.62	5.62	5.57	5.63
Median	6	6	6	6	6	6





Fig. 1. Overall rating scale for individual parameters for Faculty Feedback Rating

Weighted average with 80/20 criteria, mean and median of faculty 1 with respect to all parameters have been calculated and shown in table III. It is found that overall faculty feedback calculated based on weighted average is more reliable than mean and median.

452 TABLE IV. CATEGORIZATION OF FACULTY ON THE BASIS OF RATING OF FACULTY

Rating Condition	Faculty Category
Overall faculty rating > 5 and Overall faculty rating $< = 6$	Excellent Faculty
Overall faculty rating > 4 and Overall faculty rating $< = 5$	Good Faculty
Overall faculty rating > 3 and Overall faculty rating $< = 4$	Improvement Required
Overall faculty rating < 3	Not to be used

453 In above table IV the faculties are categorized in four groups i.e., Excellent, Good, Improvement required and Not to be used.

454 TABLE V. SUMMARY OF FACULTY AND THEIR RESPECTIVE OVERALL RATING & CATEGORIZATION

Faculty Identification	Faculty Rating	Faculty Category
F 1	5.63	Excellent Faculty
F 2	4.62	Good Faculty
F 3	4.74	Good Faculty
F 4	4.41	Good Faculty
F 5	5.66	Excellent Faculty
F 6	5.55	Excellent Faculty
F 7	5.32	Excellent Faculty
F 8	4.64	Good Faculty
F 9	5.86	Excellent Faculty

455

456 In above table V all faculties have been categorized based on representative value of Faculty Rating given by the participants.

457 3. Optimal Session Time Factor (OSTF): Adequacy of the time duration for the sessions delivered by the faculty have been calculated. Responses from the participants regarding adequacy of time allotted for sessions have been collected through feedback forms in the form of 1 (for adequacy) and 0 (for inadequacy). Optimal Session Time Factor (OSTF) for faculty has been calculated as given below:

Where, A_j^i = response for sessions i given by participant j for faculty i, where i=1 to 6 and j=1 to 21, n = Total no. of participants

464 Calculation Process:465

$$OSTF_{F_1} = \frac{(A_1^1 + A_2^1 + A_3^1 + \dots + A_{21}^1)}{21}$$

466 467

$$OSTF_{F_1} = \frac{16}{21} = 0.8$$

- 472 Optimal Session Time Factors for the sessions delivered by all faculties have been calculated and given in following table:
- 473 TABLE VI. SUMMARY OF VALUES OF OPTIMAL SESSION TIME FACTOR FOR DIFFERENT FACULTY MEMBERS

OSTF	Representative Values
OSTF _{F1}	0.8
OSTF _{F2}	0.73
OSTF _{F3}	0.8
OSTF _{F4}	1
OSTF _{F5}	0.9
OSTF _{F6}	0.9

474

475 4. *Topic Difficulty Factor (TDF):* Difficulty level of the topic in each session delivered by respective faculty has been
476 received from feedback of participants in five-point scale i.e. 5 for Very Difficult, 4 for Difficult, 3 for Moderate, 2 for Easy
477 and 1 for Very Easy. Representative values of topic difficulty level for sessions delivered by first faculty is evaluated in table
478 VII.

479

480 TABLE VII. CALCULATION PROCESS OF TOPIC DIFFICULTY FACTOR USING WEIGHTED AVERAGE W. R. T. FACULTY 1

Ratings of Parameter	Rating Parameters Values (w.r.t ratings of parameters)	Ratings of Parameter * Rating Parameters Values
3	13	39
2	6	12
1	2	2
4	0	0
5	0	0
		$TDF_{F_1} = (13*3 + 6*2)/(13+6) = 2.68$

481

Representative values of Topic Difficulty Factor for the sessions delivered by all faculties have been calculated and presented in following table:

484 TABLE VIII. SUMMARY OF REPRESENTATIVE VALUE OF TOPIC DIFFICULTY FACTOR FOR DIFFERENT FACULTY MEMBERS

TDF	Representative Values
TDF _{F1}	2.68
TDF _{F2}	2.5
TDF _{F3}	2.67
TDF _{F4}	2.57
TDF _{F5}	2.59
TDF _{F6}	2.9

485

The following table IX shows the representative learning indices of all sessions delivered by the faculty members along with representative values of all four factors affecting learning indices as calculated in previous sections:

488

489 TABLE IX. SUMMARY OF TOPIC WISE LI, FFR, OSTF & TDF

S. No	Session/Topic	Session/Topic Faculty		Faculty Feedback Rating (FFR)	Optimal Session Time Factor (OSTF)	Topic Difficulty Factor (TDF)	
1	Project Planning and Management: An Overview	F1	0.48	5.63	0.8	2.68	

$\begin{aligned} \frac{3}{\pi} & \frac{\log_2}{\log_2} & \frac{\log_2}{1} & \frac{\log_2}{1} & \frac{1}{14} & \frac{1}{0,47} & \frac{1}{5,88} & \frac{1}{1} & \frac{1}{2,57} \\ \frac{1}{5} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{5} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{5} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{5} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{5} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} &$	2	PPFM & PEARL	F2	0.57	4.59	0.73	2.5
$\frac{1}{4} = \frac{1}{16} =$	3	Project Qualit	ty F3	0.55	4.64	0.8	2.67
isInterviewisIsIsisInterviewIs0.325.660.92.59isTrue EstimationFe0.475.550.92.9A. DATA INTERPRETATION, HANDING AND ANALYSISIn order to obtain the relation between learning indices with all three factors affecting Learning Index, multivariate regranalysis has been performed as follows:For simplifying the notations in equations, following variables are considered instead of Learning Index and three factorTABLE XNERESENTITIVE VARIANCE WEIT FACTORS & Learning Index SymbolNERESENTITIVE VARIANCE WEIT FACTORS & Learning Index SymbolMULTIVE VARIANCE WEIT FACTORS & Learning Index Symbol	4	Management Team Building	F4	0.47	5.86	1	2.57
$\frac{ \mathbf{x} _{\mathbf{x}} = \frac{ \mathbf{x} _{\mathbf{x}} = \frac{ \mathbf{x} _{\mathbf{x}} = \frac{ \mathbf{x} _{\mathbf{x}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}}}{ \mathbf{x} _{\mathbf{x}}}} = \frac{ \mathbf{x} _{\mathbf{x}}$	5	Network Graphs	F5	0.52	5.66	0.0	2.57
$\frac{1}{2} (100 \text{ FSMMARO}) \qquad (10) \qquad (2) \qquad (2)$	5	Time Estimation	15	0.32	5.00	0.9	2.37
4. DATA INTERPRETATION, HANDLING AND ANALYSIS In order to obtain the relation between learning indices with all three factors affecting Learning Index, multivariate regar- analysis has been performed as follows: For simplifying the notations in equations, following variables are considered instead of Learning Index and three factor TABLEX EXERSISTATUSE VARIABLES W.R.T. PECTORS & LEARNER GREEN YAMBUES Note: Note: Not	0	Time Estimation	10	0.47	5.55	0.9	2.9
TABLE X. TABLE X. TREPRESENTATIVES VARIABLES W. R.T. FACTORS & LEARNING FORMS YMBOLS. 	In order t analysis f For simpl	o obtain the relation betw has been performed as foll ifving the notations in equi	4. DATA IN veen learning lows: uations, follo	TERPRETATION, I indices with all to owing variables an	HANDLING AND AN.	ALYSIS ng Learning Index, 1 d of Learning Index	multivariate regr
$\begin{split} & \frac{\mathbf{S} \ No. \mathbf{Factors} \ \& \ \mathbf{Learning Index} \ Symbols \frac{\mathbf{Representatives Variables}{1}}{\frac{1}{2} - \frac{1}{\mathrm{Factory} \ With real to K \ \mathrm{Kaing} \ (Re)} \frac{N1}{22} - \frac{1}{\mathrm{Factor} \ With real to K \ \mathrm{Kaing} \ (Re)} \frac{N1}{22} - \frac{1}{\mathrm{Factor} \ With real to K \ \mathrm{Kaing} \ (Re)} \frac{N2}{23} - \frac{1}{\mathrm{Sigmal} \ \mathrm{Sigmal} \ \mathrm{Sigma} \ \mathrm{Sigmal} \ \mathrm{Sigmal} \ \mathrm{Sigma} \ \mathrm{Sigmal} \ \mathrm{Sigma} \ \mathrm$	TABLE X.	REPRESENTATIVES V	ARIABLES W.R.T	. FACTORS & LEARNIN	NG INDEX SYMBOLS.	U	
$\frac{1}{2} \frac{1}{a} \frac{1}$		S. No.	Factors &	Learning Index Sym	bols Representat	tives Variables	
$\frac{1}{2} \sum_{\alpha} \sum_{\alpha} (x_{1\alpha}) \left(x_{1\alpha} - b_{1} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4} \right) \left(-1 \right) = 0$ $\sum_{\alpha} X_{1\alpha} = b_{1} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4} \right) (-1) = 0$ $\sum_{\alpha} X_{1\alpha} = b_{0} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4} \right) (-1) = 0$ For minimum error $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{\alpha} (X_{1\alpha} - b_{1} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4}) (-1) = 0$ $\sum_{\alpha} X_{1\alpha} = 6b_{1} + b_{2}\sum_{\alpha} X_{2}x_{1} + b_{3}\sum_{\alpha} X_{3} - b_{4}X_{4} \right) (-2X_{2\alpha}) = 0$ $for minimum error$ $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{\alpha} (X_{1\alpha} - b_{1} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4}) (-2X_{2\alpha}) = 0$ $\int_{\alpha} X_{1\alpha} = bb_{1}\sum_{\alpha} X_{2}x_{1} + b_{2}\sum_{\alpha} X_{2}x_{1}^{2} + b_{3}\sum_{\alpha} X_{2}x_{3}x_{1} + b_{4}\sum_{\alpha} X_{4}x_{4} \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $\sum_{\alpha} X_{1\alpha} = bb_{1}\sum_{\alpha} X_{2}x_{1} + b_{2}\sum_{\alpha} X_{2}x_{1}^{2} + b_{3}\sum_{\alpha} X_{3}x_{1} + b_{4}\sum_{\alpha} X_{4}x_{4} \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $\sum_{\alpha} (X_{1\alpha} - b_{1} - b_{2}X_{2} - b_{3}X_{3} - b_{4}X_{4}) (-2X_{2\alpha}) = 0$ $\int_{\alpha} X_{2}x_{1}x_{1} = b_{1}\sum_{\alpha} X_{2}x_{1} + b_{2}\sum_{\alpha} X_{2}x_{1}^{2} + b_{3}\sum_{\alpha} X_{2}x_{3}x_{1} + b_{4}\sum_{\alpha} X_{2}x_{4}x_{4} \dots \dots (3)$ Similar putting $\frac{m}{ab_{2}} = 0, \frac{m}{ab_{2}} = 0$ $\sum_{\alpha} X_{3}x_{1}x_{\alpha} = b_{1}\sum_{\alpha} X_{3}x_{\alpha} + b_{2}\sum_{\alpha} X_{2}x_{3}x_{\alpha} + b_{3}\sum_{\alpha} X_{2}x_{4}x_{\alpha} \dots \dots (4)$		1	Learning In	dev (LL-)	X1		
$\frac{1}{3} \qquad \frac{1}{2} = \frac{1}{2} $		1.					
$\frac{3}{4} \qquad \begin{array}{c c c c c c } \hline \begin{array}{c c c c } \hline \begin{array}{c} 33 \\ \hline $		2.	Faculty Fee	edback Rating (R _{Fi})	X2		× ·
4. Topic Difficulty Factor (TDF _x) X4 Multivariate regression equation is given as follows: $X1_{a} = b_{1} + b_{2}X2_{a} + b_{3}X3_{a} + b_{4}X4_{4}.$ Where, α is the number of faculties ($\alpha = 1, 2, 3, 4, 5, 6$), b_{1}, b_{2}, b_{3} , and b_{4} are unknown variables, X_{1}, X_{2}, X_{3} , and $k_{nown variables}.$ $Error E = \sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots (1)$ $\frac{\partial B}{\partial b_{1}} = 0$ $2\sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{\alpha} X_{1a} = 6b_{1} + b_{2}\sum_{\alpha} X2_{a} + b_{3}\sum_{\alpha} X3_{a} + b_{4}\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{\alpha} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a} + b_{4}\sum_{\alpha} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar putting $\frac{\pi b_{3}}{\pi b_{3}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{3}\sum_{\alpha} X3_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (3)$ Similar putting $\frac{\pi b_{3}}{\pi b_{3}} = 0$		3.	Optimal Se	ssion Time Factor (O	STF _{Fi}) X3		
Multivariate regression equation is given as follows: $X_{1_{a}} = b_{1} + b_{2}X_{2_{a}} + b_{3}X_{3_{a}} + b_{4}X_{4_{a}}$ Where, a is the number of facultics ($a = 1, 2, 3, 4, 5, 6$), b_{1}, b_{2}, b_{3} , and b_{4} are unknown variables, X_{1}, X_{2}, X_{3} , and known variables. $Error E = \sum_{a}^{a} (X_{1_{a}} - b_{1} - b_{2}X_{2_{a}} - b_{3}X_{3_{a}} - b_{4}X_{4_{a}})^{2} \dots \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a} (X_{1_{a}} - b_{1} - b_{2}X_{2_{a}} - b_{3}X_{3_{a}} - b_{4}X_{4_{a}})(-1) = 0$ $\sum_{a} X_{1_{a}} = 6b_{1} + b_{2}\sum_{a} X_{2_{a}} + b_{3}\sum_{a} X_{3_{a}} + b_{4}\sum_{a} X_{4_{a}} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a} (X_{1_{a}} - b_{1} - b_{2}X_{2_{a}} - b_{3}X_{3_{a}} - b_{4}X_{4_{a}})(-2X_{2_{a}}) = 0$ $-4\sum_{a} (X_{1_{a}}X_{2_{a}} - b_{1}X_{2_{a}} - b_{2}X_{2_{a}}^{2} - b_{3}X_{3_{a}}X_{2_{a}} - b_{a}X_{4_{a}}X_{2_{a}}) = 0$ $\sum_{a} X_{2_{a}}X_{1_{a}} = b_{1}\sum_{a} X_{2_{a}} + b_{2}\sum_{a} X_{2_{a}}^{2} + b_{3}\sum_{a} X_{2_{a}}X_{3_{a}} + b_{4}\sum_{a} X_{2_{a}}X_{4_{a}} \dots \dots \dots (3)$ Similar putting $\frac{w}{a_{3_{a}}} = 0, \frac{w}{a_{3_{a}}} = 0$ $\sum_{a} X_{3_{a}}X_{1_{a}} = b_{1}\sum_{a} X_{3_{a}} + b_{2}\sum_{a} X_{2_{a}}X_{3_{a}} + b_{3}\sum_{a} X_{3_{a}}^{2} + b_{4}\sum_{a} X_{4_{a}}X_{4_{a}} \dots \dots \dots (4)$		4.	Topic Diffi	culty Factor (TDF _{Fi})	X4		
Multivariate regression equation is given as follows: $x1_{a} = b_{1} + b_{2}X2_{a} + b_{3}X3_{a} + b_{4}X4_{a}$ Where, a is the number of faculties ($a = 1, 2, 3, 4, 5, 6$), b_{1}, b_{2}, b_{3} , and b_{4} are unknown variables, X_{1}, X_{2}, X_{3} , and k known variables. $Error E = \sum_{a}^{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a}^{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a}^{a} X1_{a} = 6b_{1} + b_{2}\sum_{a}^{a} X2_{a} + b_{3}\sum_{a}^{a} X3_{a} + b_{4}\sum_{a}^{a} X4_{a} \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a}^{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2}a) = 0$ $-4\sum_{a}^{a} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a}X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a}^{a} X2_{a}X1_{a} = b_{1}\sum_{a}^{a} X2_{a} + b_{2}\sum_{a}^{a} X2_{a}^{2} + b_{3}\sum_{a}^{a} X2_{a}X3_{a} + b_{4}\sum_{a}^{a} X2_{a}X4_{a} \dots \dots (3)$ Similar parting $\frac{de}{db_{3}} = 0, \frac{de}{db_{3}} = $		L			1		
$\begin{aligned} x_{1a} = b_1 + b_2 X_a^2 + b_3 X_a^2 + b_4 X_a^4 \\ \text{Where, } a \text{ is the number of faculties } (a = 1, 2, 3, 4, 5, 6), b_1, b_2, b_3, \text{ and } b_4 \text{ are unknown variables, } X_1, X_2, X_3, \text{ and known variables.} \\ Error E = \sum_{a}^{-1} (X_{1a} - b_1 - b_2 X_{2a}^2 - b_3 X_{3a}^2 - b_4 X_{4a})^2 \dots \dots \dots (1) \\ \frac{\partial E}{\partial b_1} = 0 \\ 2\sum_{a}^{-1} (X_{1a} + b_1 - b_2 X_{2a}^2 - b_3 X_{3a}^2 - b_4 X_{4a})(-1) = 0 \\ \sum_{a}^{-1} X_{1a}^2 = 6b_1 + b_2 \sum_{a}^{-1} X_{2a}^2 + b_3 \sum_{a}^{-1} X_{3a}^2 + b_4 \sum_{a}^{-1} X_{4a}^2 \dots \dots \dots (2) \end{aligned}$ For minimum error $\frac{\partial E}{\partial b_2} = 0 \\ 2\sum_{a}^{-1} (X_{1a} - b_1 - b_2 X_{2a}^2 - b_3 X_{3a}^2 - b_4 X_{4a})(-2X_{2a}) = 0 \\ -4\sum_{a}^{-1} (X_{1a} X_{2a}^2 - b_2 X_{2a}^2 - b_3 X_{3a}^2 - b_4 X_{4a})(-2X_{2a}) = 0 \\ \sum_{a}^{-1} X_{2a}^2 X_{1a}^2 = b_1 \sum_{a}^{-1} X_{2a}^2 + b_2 \sum_{a}^{-1} X_{2a}^2 + b_3 \sum_{a}^{-1} X_{2a}^2 X_{2a}^2 + b_4 \sum_{a}^{-1} X_{2a}^2 X_{4a}^2 \dots \dots \dots (3) \end{aligned}$ Similar puting $\frac{\partial E}{\partial b_2} = 0$ $\sum_{a}^{-1} X_{2a}^2 X_{1a}^2 = b_1 \sum_{a}^{-1} X_{2a}^2 + b_2 \sum_{a}^{-1} X_{2a}^2 + b_3 \sum_{a}^{-1} X_{2a}^2 X_{2a}^2 + b_4 \sum_{a}^{-1} X_{2a}^2 X_{4a}^4 \dots \dots \dots (3)$	Multivar	iate regression equation is	s given as fol	llows:			
$X1_{a} = b_{1} + b_{2}X2_{a} + b_{3}X3_{a} + b_{4}X4_{a}$ Where, <i>a</i> is the number of faculties (<i>a</i> = 1, 2, 3, 4, 5, 6), <i>b</i> ₁ , <i>b</i> ₂ , <i>b</i> ₃ , and <i>b</i> ₄ are unknown variables, X ₁ , X ₂ , X ₃ , and known variables. Error $E = \sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a}^{\infty} X1_{a} = 6b_{1} + b_{2}\sum_{a}^{\infty} X2_{a} + b_{3}\sum_{a}^{\infty} X3_{a} + b_{4}\sum_{a}^{\infty} X4_{a} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a}^{\infty} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a} X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a}^{\infty} X2_{a}X1_{a} = b_{1}\sum_{a}^{\infty} X2_{a} + b_{2}\sum_{a}^{\infty} X2_{a}^{2} + b_{3}\sum_{a}^{\infty} X2_{a}X3_{a} + b_{4}\sum_{a}^{\infty} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar parting $\frac{de_{ay}}{dy_{a}} = 0, \frac{dy_{a}}{dy_{a}} = 0$ $\sum_{a}^{\infty} X3_{a}X1_{a} = b_{1}\sum_{a}^{\infty} X3_{a} + b_{2}\sum_{a}^{\infty} X2_{a}X3_{a} + b_{3}\sum_{a}^{\infty} X3_{a}^{2} + b_{4}\sum_{a}^{\infty} X4_{a}X3_{a} \dots \dots (4)$	manival	ine regression equation is	5 51 011 do 101				
Where, a is the number of faculties (a = 1, 2, 3, 4, 5, 6), b ₁ , b ₂ , b ₃ , and b ₄ are unknown variables, X ₁ , X ₂ , X ₃ , and known variables. $Error E = \sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a}^{\infty} X1_{a} = 6b_{1} + b_{2}\sum_{a}^{\infty} X2_{a} + b_{3}\sum_{a}^{\infty} X3_{a} + b_{4}\sum_{a}^{\infty} X4_{a} \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a}^{\infty} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a}^{\infty} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $\sum_{a}^{\infty} X2_{a}X1_{a} = b_{1}\sum_{a}^{\infty} X2_{a} + b_{2}\sum_{a}^{\infty} X2_{a}^{2} + b_{3}\sum_{a}^{\infty} X2_{a}X3_{a} + b_{4}\sum_{a}^{\infty} X2_{a}X4_{a} \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{2}} = 0$ $\sum_{a}^{\infty} X3_{a}X1_{a} = b_{1}\sum_{a}^{\infty} X3_{a} + b_{2}\sum_{a}^{\infty} X2_{a}^{2} + b_{3}\sum_{a}^{\infty} X2_{a}X3_{a} + b_{4}\sum_{a}^{\infty} X2_{a}X4_{a} \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{2}} = 0$			<i>X</i> 1	$a_1 = h_1 + h_2 X^2 + h_1^2$	$h_2 X_{3_m} + h_2 X_4$		
Where, α is the number of faculties ($\alpha = 1, 2, 3, 4, 5, 6$), $b_1, b_2, b_3, and b_4$ are unknown variables, X_1, X_2, X_3 , and known variables. $Error E = \sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})^2 \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_1} = 0$ $2 \sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_1 + b_2 \sum_{\alpha} X2_{\alpha} + b_3 \sum_{\alpha} X3_{\alpha} + b_4 \sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2 \sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4 \sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1 \sum_{\alpha} X2_{\alpha} + b_2 \sum_{\alpha} X2_{\alpha}^2 + b_3 \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4 \sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{dE}{db_1} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1 \sum_{\alpha} X3_{\alpha} + b_2 \sum_{\alpha} X3_{\alpha}^2 + b_3 \sum_{\alpha} X3_{\alpha}^2 + b_4 \sum_{\alpha} X4_{\alpha}X3_{\alpha} + \dots \dots (3)$			A 1	$\alpha = b_1 + b_2 \Lambda L_{\alpha} + b_{\alpha}$	$J_3 \dots J_{\alpha} = D_4 \dots T_{\alpha}$		
known variables. $Error E = \sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a}^{c} X1_{a} = 6b_{1} + b_{2}\sum_{a}^{c} X2_{a} + b_{3}\sum_{a}^{c} X3_{a} + b_{4}\sum_{a}^{c} X4_{a} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a}^{c} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a} X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a}^{c} X2_{a}X1_{a} = b_{1}\sum_{a}^{c} X2_{a} + b_{2}\sum_{a}^{c} X2_{a}^{2} + b_{3}\sum_{a}^{c} X2_{a}X3_{a} + b_{4}\sum_{a}^{c} X2_{a}X4_{a} \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0$ $\sum_{a}^{c} X3_{a}X1_{a} = b_{1}\sum_{a}^{c} X3_{a} + b_{2}\sum_{a}^{c} X2_{a}^{2}X3_{a} + b_{3}\sum_{a}^{c} X3_{a}^{2} + b_{4}\sum_{a}^{c} X4_{a}X3_{a} \dots \dots (4)$	Where, α	is the number of faculti	es ($\alpha = 1, 2,$	3, 4, 5, 6), b ₁ , t	b_2 , b_3 , and b_4 are u	nknown variables. X	X_1, X_2, X_3 , and Σ
$Error E = \sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a} X1_{a} = 6b_{1} + b_{2}\sum_{a} X2_{a} + b_{3}\sum_{a} X3_{a} + b_{4}\sum_{a} X4_{a} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a} X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a} X2_{a}X1_{a} = b_{1}\sum_{a} X2_{a} + b_{2}\sum_{a} X2_{a}^{2} + b_{3}\sum_{a} X2_{a}X3_{a} + b_{4}\sum_{a} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{1}} = 0$ $\sum_{a} X3_{a}X1_{a} = b_{1}\sum_{a} X3_{a} + b_{2}\sum_{a} X2_{a}^{2} + b_{3}\sum_{a} X3_{a}^{2} + b_{4}\sum_{a} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{1}} = 0$	known va	riables.	())		27 - 57	· · · · · · · · · · · · · · · ,	17 27 37
$Error E = \sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})^{2} \dots \dots \dots \dots (1)$ $\frac{\partial E}{\partial b_{1}} = 0$ $2\sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a}^{c} X1_{a} = 6b_{1} + b_{2}\sum_{a}^{c} X2_{a} + b_{3}\sum_{a}^{c} X3_{a} + b_{4}\sum_{a}^{c} X4_{a} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a}^{c} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a}^{c} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a}X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a}^{c} X2_{a}X1_{a} = b_{1}\sum_{a}^{c} X2_{a} + b_{2}\sum_{a}^{c} X2_{a}^{2} + b_{3}\sum_{a}^{c} X2_{a}X3_{a} + b_{4}\sum_{a}^{c} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar putting $\frac{dE}{db_{1}} = 0$ $\sum_{a}^{c} X3_{a}X1_{a} = b_{1}\sum_{a}^{c} X3_{a} + b_{2}\sum_{a}^{c} X2_{a}X3_{a} + b_{3}\sum_{a}^{c} X3_{a}^{2} + b_{4}\sum_{a}^{c} X4_{a}X3_{a} \dots \dots (4)$			_				
$\frac{dE}{db_1} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_1 + b_2\sum_{\alpha} X2_{\alpha} + b_3\sum_{\alpha} X3_{\alpha} + b_4\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\delta E}{\delta b_1} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$		Er	$ror E = \sum'$	$(X1_{\alpha} - b_1 - b_2 X2_{\alpha})$	$-b_3 X 3_\alpha - b_4 X 4_\alpha)^2$		
$\frac{\partial E}{\partial b_1} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_1 + b_2\sum_{\alpha} X2_{\alpha} + b_3\sum_{\alpha} X3_{\alpha} + b_4\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_2} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$			$\frac{2}{\alpha}$				
$\frac{\partial E}{\partial b_1} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_1 + b_2\sum_{\alpha} X2_{\alpha} + b_3\sum_{\alpha} X3_{\alpha} + b_4\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{dE}{db_3} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X3_{\alpha}^2$				ar			
$2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_{1} + b_{2}\sum_{\alpha} X2_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha} + b_{4}\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$				$\frac{\partial E}{\partial L} =$	= 0		
$2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-1) = 0$ $\sum_{\alpha} X1_{\alpha} = 6b_{1} + b_{2}\sum_{\alpha} X2_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha} + b_{4}\sum_{\alpha} X4_{\alpha} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$				∂b_1	7		
$2\sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-1) = 0$ $\sum_{a} X1_{a} = 6b_{1} + b_{2}\sum_{a} X2_{a} + b_{3}\sum_{a} X3_{a} + b_{4}\sum_{a} X4_{a} \dots \dots \dots (2)$ For minimum error $\frac{\partial E}{\partial b_{2}} = 0$ $2\sum_{a} (X1_{a} - b_{1} - b_{2}X2_{a} - b_{3}X3_{a} - b_{4}X4_{a})(-2X_{2a}) = 0$ $-4\sum_{a} (X1_{a}X2_{a} - b_{1}X2_{a} - b_{2}X2_{a}^{2} - b_{3}X3_{a}X2_{a} - b_{4}X4_{a}X2_{a}) = 0$ $\sum_{a} X2_{a}X1_{a} = b_{1}\sum_{a} X2_{a} + b_{2}\sum_{a} X2_{a}^{2} + b_{3}\sum_{a} X2_{a}X3_{a} + b_{4}\sum_{a} X2_{a}X4_{a} \dots \dots \dots (3)$ Similar putting $\frac{B}{b_{3}} = 0$ $\sum_{a} X3_{a}X1_{a} = b_{1}\sum_{a} X3_{a} + b_{2}\sum_{a} X2_{a}X3_{a} + b_{3}\sum_{a} X3_{a}^{2} + b_{4}\sum_{a} X4_{a}X3_{a} \dots \dots \dots (4)$							
For minimum error $ \sum_{\alpha} X1_{\alpha} = 6b_{1} + b_{2} \sum_{\alpha} X2_{\alpha} + b_{3} \sum_{\alpha} X3_{\alpha} + b_{4} \sum_{\alpha} X4_{\alpha} \dots \dots \dots (2) $ For minimum error $ \frac{\partial E}{\partial b_{2}} = 0 $ $ 2 \sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0 $ $ -4 \sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0 $ $ \sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} X2_{\alpha} + b_{2} \sum_{\alpha} X2_{\alpha}^{2} + b_{3} \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4} \sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) $ Similar putting $ \frac{\partial E}{\partial b_{3}} = 0 $ $ \sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} X3_{\alpha} + b_{2} \sum_{\alpha} X2_{\alpha}^{2} + b_{3} \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4} \sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (4) $			$2\sum (X$	$(1_{\alpha} - b_1 - b_2 X 2_{\alpha} - b_2 X 2_{\alpha})$	$b_2 X 3_{\alpha} - b_4 X 4_{\alpha} (-1)$	0 = 0	
For minimum error $\begin{aligned} \sum_{\alpha} X1_{\alpha} &= 6b_1 + b_2 \sum_{\alpha} X2_{\alpha} + b_3 \sum_{\alpha} X3_{\alpha} + b_4 \sum_{\alpha} X4_{\alpha} \dots \dots \dots (2) \\ & \qquad \qquad$			\sum_{α}	u 11 12 u	-5-u -4 u/C)		
For minimum error $ \begin{aligned} \sum_{\alpha} & X1_{\alpha} = 6b_{1} + b_{2} \sum_{\alpha} & X2_{\alpha} + b_{3} \sum_{\alpha} & X3_{\alpha} + b_{4} \sum_{\alpha} & X4_{\alpha} \dots \dots \dots (2) \\ & & & \\ \frac{\partial E}{\partial b_{2}} = 0 \\ & & 2\sum_{\alpha} & (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0 \\ & & -4\sum_{\alpha} & (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0 \\ & & \sum_{\alpha} & X2_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} & X2_{\alpha} + b_{2} \sum_{\alpha} & X2_{\alpha}^{2} + b_{3} \sum_{\alpha} & X2_{\alpha}X3_{\alpha} + b_{4} \sum_{\alpha} & X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) \end{aligned} $ Similar putting $\frac{\partial E}{\partial b_{2}} = 0 \\ & \sum_{\alpha} & X2_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} & X2_{\alpha} + b_{2} \sum_{\alpha} & X2_{\alpha}^{2} + b_{3} \sum_{\alpha} & X2_{\alpha}X3_{\alpha} + b_{4} \sum_{\alpha} & X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) \end{aligned} $ Similar putting $\frac{\partial E}{\partial b_{2}} = 0 \\ & \sum_{\alpha} & X2_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} & X3_{\alpha} + b_{2} \sum_{\alpha} & X2_{\alpha}X3_{\alpha} + b_{3} \sum_{\alpha} & X3_{\alpha}^{2} + b_{4} \sum_{\alpha} & X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$							
For minimum error $ \begin{aligned} \sum_{\alpha} X1_{\alpha} &= 6b_{1} + b_{2} \sum_{\alpha} X2_{\alpha} + b_{3} \sum_{\alpha} X3_{\alpha} + b_{4} \sum_{\alpha} X4_{\alpha} \dots \dots \dots (2) \\ & \frac{\partial E}{\partial b_{2}} = 0 \\ & 2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0 \\ & -4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0 \\ & \sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1} \sum_{\alpha} X2_{\alpha} + b_{2} \sum_{\alpha} X2_{\alpha}^{2} + b_{3} \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4} \sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$							
For minimum error $ \frac{\partial E}{\partial b_2} = 0 $ $ 2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0 $ $ -4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0 $ $ -4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0 $ $ \sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) $ Similar putting $ \frac{\partial E}{\partial b_2} = 0 $ $ \sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3) $ Similar putting $ \frac{\partial E}{\partial b_2} = 0 $ $ \sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4) $		∇		<u>v</u> v	\sim		
For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$		$\sum_{i=1}^{n}$	$X1_{\alpha} = 6b_1 + b_1$	$X2_{\alpha} + b_3$	$X3_{\alpha} + b_4$	$X4_{\alpha}$ (2)	
For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_2} = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$		$\frac{1}{\alpha}$			αα		
For minimum error $\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_2} = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$							
$\frac{\partial E}{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$	For minimu	m error					
$\overline{\partial b_2} = 0$ $2\sum_{\alpha} (X1_{\alpha} - b_1 - b_2X2_{\alpha} - b_3X3_{\alpha} - b_4X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_1X2_{\alpha} - b_2X2_{\alpha}^2 - b_3X3_{\alpha}X2_{\alpha} - b_4X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X2_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}^2 + b_3\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_4\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1\sum_{\alpha} X3_{\alpha} + b_2\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_3\sum_{\alpha} X3_{\alpha}^2 + b_4\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$. or minimu		_	∂E	_		
$2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$				$\frac{\partial D}{\partial b_2} =$	= 0		
$2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$				<u>Z</u>			
$2\sum_{\alpha} (X1_{\alpha} - b_{1} - b_{2}X2_{\alpha} - b_{3}X3_{\alpha} - b_{4}X4_{\alpha})(-2X_{2\alpha}) = 0$ $-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$			$\sim \Sigma$				
$-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$			$^{2}\sum (X1)$	$_{\alpha}-b_{1}-b_{2}X2_{\alpha}-b_{1}$	$b_3 X \mathcal{Z}_\alpha - b_4 X \mathcal{Z}_\alpha)(-2X_2$	$(\alpha) = 0$	
$-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$			α				
$-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$		Y					
$-4\sum_{\alpha} (X1_{\alpha}X2_{\alpha} - b_{1}X2_{\alpha} - b_{2}X2_{\alpha}^{2} - b_{3}X3_{\alpha}X2_{\alpha} - b_{4}X4_{\alpha}X2_{\alpha}) = 0$ $\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$			$\mathbf{\nabla}$				
$\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$			$4\sum (X1_{\alpha}X2)$	$a_{\alpha} - b_1 X 2_{\alpha} - b_2 X 2_{\alpha}^2$	$a_{\alpha}^2 - b_3 X 3_{\alpha} X 2_{\alpha} - b_4 X^2$	$4_{\alpha} X 2_{\alpha}) = 0$	
$\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$			α				
$\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$							
$\sum_{\alpha} X2_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X2_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}^{2} + b_{3}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{4}\sum_{\alpha} X2_{\alpha}X4_{\alpha} \dots \dots \dots (3)$ Similar putting $\frac{\partial E}{\partial b_{3}} = 0, \frac{\partial E}{\partial b_{4}} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_{1}\sum_{\alpha} X3_{\alpha} + b_{2}\sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_{3}\sum_{\alpha} X3_{\alpha}^{2} + b_{4}\sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots (4)$							
$\sum_{\alpha} X_{\alpha} X_{\alpha} = b_{1} \sum_{\alpha} X_{\alpha} + b_{2} \sum_{\alpha} X_{\alpha} + b_{3} \sum_{\alpha} X_{\alpha} X_{\alpha} + b_{4} \sum_{\alpha} X_{\alpha} X_{\alpha} + b_$			\sum_{k}	$\sum v^2$			(2)
$\alpha \qquad \alpha \qquad \alpha \qquad \alpha \qquad \alpha \qquad \overline{\alpha}$ Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1 \sum_{\alpha} X3_{\alpha} + b_2 \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_3 \sum_{\alpha} X3_{\alpha}^2 + b_4 \sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$		$\sum X_{\alpha}^{2} X_{\alpha}^{1} = 1$	$\nu_1 \Delta X^2_{\alpha}$	$+ p_2 \sum X Z_{\alpha}^2 + k$	$p_3 \angle X_{\alpha}^{X_{\alpha}} + b_4$		(3)
Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum X3_{\alpha}X1_{\alpha} = b_1 \sum X3_{\alpha} + b_2 \sum X2_{\alpha}X3_{\alpha} + b_3 \sum X3_{\alpha}^2 + b_4 \sum X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$		α	α	α	α	α	
Similar putting $\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum X3_{\alpha}X1_{\alpha} = b_1 \sum X3_{\alpha} + b_2 \sum X2_{\alpha}X3_{\alpha} + b_3 \sum X3_{\alpha}^2 + b_4 \sum X4_{\alpha}X3_{\alpha} \dots \dots \dots (4)$							
$\frac{\partial E}{\partial b_3} = 0, \frac{\partial E}{\partial b_4} = 0$ $\sum_{\alpha} X3_{\alpha}X1_{\alpha} = b_1 \sum_{\alpha} X3_{\alpha} + b_2 \sum_{\alpha} X2_{\alpha}X3_{\alpha} + b_3 \sum_{\alpha} X3_{\alpha}^2 + b_4 \sum_{\alpha} X4_{\alpha}X3_{\alpha} \dots \dots \dots \dots (4)$	Similar putt	ing					
$\sum_{ab_3} X_{ab_4} = b_1 \sum_{ab_4} X_{ab_4} = b_1 \sum_{ab_4} X_{ab_4} + b_2 \sum_{ab_4} X_{ab_4} + b_3 \sum_{ab_4} X_{ab_4} \sum_{ab_4} X_{ab_4} X_{ab_4} + b_4 \sum_{ab_4} X_{ab_4} + b_4 \sum$	$\frac{\partial E}{\partial E} = 0 \frac{\partial E}{\partial E}$	-= 0					
$\sum X3_{\alpha}X1_{\alpha} = b_1 \sum X3_{\alpha} + b_2 \sum X2_{\alpha}X3_{\alpha} + b_3 \sum X3_{\alpha}^2 + b_4 \sum X4_{\alpha}X3_{\alpha} \dots \dots \dots \dots (4)$	$\partial b_3 = 0, \ \partial b_4$.					
$\sum X_{\alpha}X_{1_{\alpha}} = b_{1} \sum X_{\alpha} + b_{2} \sum X_{\alpha}X_{\alpha} + b_{3} \sum X_{\alpha}X_{\alpha}^{2} + b_{4} \sum X_{4_{\alpha}}X_{3_{\alpha}} \dots \dots \dots \dots (4)$		∇	∇	∇	$\sum_{i=1}^{n}$	∇	
		$\sum X3_{\alpha}X1_{\alpha} = 1$	$b_1 $ $X3_{\alpha}$	$+ b_2 \sum X2_{\alpha}X3_{\alpha}$	$+ b_3 \sum X3_{\alpha}^2 + b_4$	$X4_{\alpha}X3_{\alpha}\dots\dots\dots$	(4)

$$\sum_{\alpha} \quad X4_{\alpha}X1_{\alpha} = b_1 \sum_{\alpha} \quad X4_{\alpha} + b_2 \sum_{\alpha} \quad X2_{\alpha}X4_{\alpha} + b_3 \sum_{\alpha} \quad X3_{\alpha}X4_{\alpha} + b_4 \sum_{\alpha} \quad X4_{\alpha}^2 \dots \dots \dots \dots (5)$$

522 523 524 Equation (2) can be written as

520 521

533

536

	• • • •	$6\overline{X1} = 6b_1 + 6b_2\overline{X2} + 6b_2\overline{X3} + 6b_4\overline{X4}$
525		1 <i>L</i> J T
526		
		$\overline{X1} = b_1 + b_2 \overline{X2} + b_3 \overline{X3} + b_4 \overline{X4} \dots \dots \dots \dots \dots \dots (A)$
527		
528	From equation (3)	
529		$\overline{X1X2} = b_1\overline{X2} + b_2\overline{X2^2} + b_3\overline{X2X3} + b_4\overline{X2X4} \dots \dots \dots \dots \dots (B)$
530	From equation (4)	
		$\overline{X1X3} = b_1\overline{X3} + b_2\overline{X2X3} + b_3\overline{X3^2} + b_4\overline{X3X4} \dots \dots$
531		
532	From equation (5)	
		$\overline{X1X4} = b_1 \overline{X4} + b_2 \overline{X2X4} + b_3 \overline{X3X4} + b_4 \overline{X4} \dots \dots$

Above linear equations can be represented in following matrix form:

Γ	1 X2	ХЗ	X4]	[b1]		ΛI	
$\overline{X2}$	$\overline{X2^2}$	<u>X2X3</u>	$\overline{X2X4}$	b2	_	$\overline{X1X2}$	
$\overline{X3}$	<u>X2X3</u>	$\overline{X3^2}$	<u>X3X4</u>	b3	_	$\overline{X1X3}$	
$\overline{X4}$	<u>X2X4</u>	X3X	$\overline{4} \overline{X4^2}$	b4		$\left \frac{1}{X1X4} \right $	

537 In previous research, mathematical methods like curve fitting, correlation, linear equation have been used by researchers for 538 calculating True Learning. In this study, the above mathematical methods have been employed to find the best correlation. 539 Four Learning Factors are represented by linear equation which have been solved by Gauss Seidel method for calculating True 540 Learning. The matrix provided above is used to solve the equation, the values being displayed in Table XI.

541 TABLE XI. CALCULATED VALUES FOR MATRIX PARAMETERS USED IN CORRELATION.

Factor	X1	X2	X3	X4	X2 ²	X3 ²	X4 ²	X1X2	X1X3	X1X4	X2X3	X2X4	X3X4	Calculated
S. No.														Learning
1.	0.48	5.63	0.8	2.68	31.7	0.64	7.18	2.7	0.38	1.29	4.5	15.09	2.14	0.52
2.	0.57	4.59	0.73	2.5	21.07	0.53	6.25	2.62	0.42	1.43	3.35	11.48	1.83	0.53
3.	0.52	4.64	0.8	2.67	21.53	0.64	7.13	2.55	0.44	1.47	3.71	12.39	2.14	0.53
4.	0.47	5.86	1	2.57	34.34	1	6.6	2.75	0.47	1.21	5.86	15.06	2.57	0.47
5.	0.52	5.66	0.9	2.59	32.04	0.81	6.71	2.94	0.47	1.35	5.09	14.66	2.33	0.49
6.	0.47	5.55	0.9	2.9	30.8	0.81	8.41	2.61	0.42	1.36	5	16.1	2.61	0.51
Calculated Mean for each factor	0.51	5.32	0.86	2.65	28.58	0.74	7.05	2.7	0.43	1.35	4.59	14.13	2.27	

Representation of the values of True Learning & Calculated Learning and Correlation between them given below in table XII
 & Fig.3 respectively.

545 & Tig.5 Tespective

TABLE XII. SUMMARY OF TRUE AND CALCULATED LEARNING INDICES

	Y	

True Learning	Calculated Learning
0.48	0.52
0.57	0.53
0.55	0.53
0.47	0.47
0.52	0.49
0.47	0.51

545



549 This study has been able to establish a relationship between variables captured at the reaction level, from training participants, 550 from which Learning Index can be estimated, and the actual Learning Index calculated at the learning level from performance 551 of training participants at pre and post evaluation stages.

552 There is a significant amount of correlation noted from the True and Calculated Learning Indices, which implies that, by 553 capturing honest reactions from participants, learning indices of participants can be predicated to a significant degree of

554 accuracy.

546 547

548

555

560

6. DIRECTION FOR FURTHER RESEARCH

Result may be improved for that training program in which number of lectures are more (as in case of 3- or 4-week training 556

programs) and number of participants are more. Moreover, done faculty should be associated with one lecture session only. 557

558 Furthermore, for further betterment of results, number of questions in pre and post training evaluation/test should be more

559 with respect to each faculty member.

REFERENCES

- Alias, S.A., Mohd Ong, H.A. et.al (2019), "The Role of Training Design Factors in influencing Training Effectiveness among Public service Employees", International of Journal of Academic Research in Business & Social Sciences, Vol. 09, No. 05, May 2019, p.p. 898-913. E-ISSN 2222-6990, @2019 HRMARS. 561 562 563 564 565 566
 - Al-Mzary, M. M. M., Al-rifai, A. D., Al-Momany, M. O. E. (2015), "Training and Its Impact on the Performance of Employees at Jordanian Universities from the Perspective of Employees: The Case of Yarmouk University", Journal of Education and Practice, 6(32), 128-140.
 - Angela, R. L. (2017), "Interpreting and using student rating data: Guidance for faculty serving as administrators and on evaluation committees", AR. Lines/ studies in Educational Evaluation 54 (2017), 94-106. www.elsevier.com/studuc.
- 567 568 Bharthvajan, R., Kavitha, S. F. (2019), "A Research on Effectiveness of Training and Development in its Solutions", training, 12, 11. 569 570
 - Bhanji, F., Gottesman, R., de Grave, W., Steinert, Y., Winer, L. R. (2012), "The retrospective pre-post: A practical method to evaluate learning from an educational program. Academic emergency medicine", 19(2), 189-194.
 - Borate, N.S., Krishna, Dr. G., et. al. (2014), "A case study approach for evaluation of Employee Training Effectiveness and Development Program", The International Journal of Business & management, Vol. 02, Issue 06, p.p. 201-210, June 2014, ISSN 2321-8916. www.theijbm.com.
 - Borate, N.S., Gopalkrishna, Borate, S.L. (2014), "A case study approach for evaluation of Employee Training Effectiveness and Development Program", Proceeding of the Second International Conference on Global Business, Economics, Finance and Social Sciences (GB14 Conference), Chennai-India, 11-13 July 2014, Paper ID- C432. www.globalbizresearch.org.
 - Bagul, D.B. (2014), "A Research paper on 'To study the Effectiveness of Employees Training & Development Programs' ", Scholarly research Journal for Humanity Science & English Language, Vol. I, Issue IV, June July 2014, p.g. 596-603. www.srjis.com.
 - Clayson, D.E. (2009), "Student Evaluations of Teaching: Are they related to What student Learn? A Meta- Analysis and Review of the Literature", Journal of Marketing Education, Vol. 31, Number 01, April 2009, p.p. 16-30. @2009 SAGE Publications. http://jmd.sagepub.com.
 - Chahal, A. (2013), "A study of training need analysis-based training and development: Effect of training on performance by adopting development-based strategy", International Journal of Business and Management Invention, 2(4), 41-51.
- 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 588 589 590 591 Choudhury, G. B., Sharma, V. (2019), "Review and comparison of various training effectiveness evaluation models for R & D organisation performance", PM World Journal, Vol. 08, Issue 02, February 2019. www.pmworldjournal.net.
 - Heydari, M. R., Taghva, F., Amini, M., Delavari, S. (2019), "Using Kirkpatrick's model to measure the effect of a new teaching and learning methods workshop for health care staff", BMC research notes, 12(1), 1-5.
- Jonny (2016), "Training Effectiveness at PT XYZ using Kirkpatrick Model and Return on Investment of Training (ROI-Training)", Binus Business Review, Vol. 07, No. 02, p.p. 137-141. http://dx.doi.org/10.21512/bbr.v7i2.1728. 592 593

- Kanango, J., Bhatnagar, A., Gupta, R. and Kashyap, V. (2023), "Designing a short-term training course curriculum using the quality function deployment (QFD)", International Journal of Quality & Reliability Management, Vol. 40 No. 9, pp. 2247-2277. https://doi.org/10.1108/IJQRM-05-2022-0150
- Kashif, A. R., Shafiq, M., Tahir, A. H., Wahid, S., Ahmed, S. (2020), "Impact of on job training, training design and training delivery style on organizations performance", Foundation University Journal of Business & Economics, 5(2), 40-51.
- 599 600 Müller, C., Stahl, M., Alder, M., Müller, M. (2018), "Learning effectiveness and students' perceptions in a flexible learning course", European Journal of Open, Distance and E-Learning, 21(2), 44-52. 601
 - Moody, D. L., Sindre, G. (2003), "Evaluating the effectiveness of learning interventions: an information systems case study".
 - Munna, A. S., Kalam, M. A. (2021), "Teaching and learning process to enhance teaching effectiveness: a literature review", International Journal of Humanities and Innovation (IJHI), 4(1), 1-4.
 - Mehale, K. D., Govender, C. M., Mabaso, C. M. (2021), "Maximising training evaluation for employee performance improvement", SA Journal of Human Resource Management, 19, 11.
 - Rao, D.S., Vijaya, K.P. (2017), "Evaluating of Training Effectiveness based on Reaction- A case study", International Journal of Business and General Management (IJBGM), Vol. 06, Issue 04, Jun-Jul 2017, p.p. 45-56.
 - Suresh, K.C., Agrawal, M.R., Rao, R. KVS (2014), "Analysis of Effectiveness of Employee's Training in an Automotive Component Manufacturing Organisation", Research Journali's Journal of Management, Vol. 02, Issue 08, September 2014, ISSN 2347-8217.
 - Sal, A., Raja, M. (2016), "The impact of training and development on employees performance and productivity", International Journal of Management Sciences and Business Research, 5(7).

 - Saha, J. (2017), "Comparative study of Training Effectiveness Measurement Models", Global Journal of Engineering Science and Research Management, Vol. 04, Issue 12, p.p. 34-39, December 2017, ISSN 2349-4506. http://www.gjesrm.com.
 Shivaraju, P. T., Manu, G., Vinaya, M., Savkar, M. K. (2017), "Evaluating the effectiveness of pre-and post-test model of learning in a medical school", National Journal of Physiology, Pharmacy and Pharmacology, 7(9), 947.
- 617 618 Sanyal, S., Hisam, M. W. (2018), "Impact of training and development on the performance of employees-A comparative study on select banks in sultanate of Oman", International journal of scientific research and management, 6(03), 191-198.
- Theobald, M. (2021), "Self-regulated learning training programs enhance university students' academic performance, self-regulated learning strategies, and motivation: A meta-analysis", Contemporary Educational Psychology, 66, 101976. $\begin{array}{c} 619 \\ 620 \\ 621 \\ 622 \\ 623 \\ 624 \\ 625 \\ 626 \end{array}$
 - Tomic, W. (1991), "Training Programs in Research into the Effectiveness of Teacher Behavior", Journal of Education for Teaching, Vol. 17, No. 02, 1991, p.p. 181-188.
 - Zheng, L. Fluang, R., Yu, J. (2013), "Evaluation of the Effectiveness of e-Training: A case study on In-service Teacher's Training", 2013 IEEE 13th International Conference on Advance Learning Technologies, IEEE Computer Society, July 2013.

594 595 596

597 598

602 603

604 605

606 607

612