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RESEARCH ARTICLE

Effect of Suspended Particles and Magnetic Field on Thermal Convection in Ferromagnetic Fluid with Varying Gravitational Field in Porous Medium.

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Abstract

The aim of this paper is to study the effect of magnetic field and suspended particles on thermal convection in ferromagnetic fluid with varying gravity field saturating in a porous medium. A linear stability analysis and normal mode analysis methods are used to find the exact solution for a ferromagnetic fluid layer contained between two free boundaries. A dispersion relation governing the effect of magnetic field, suspended particles and medium permeability is derived theoretically. From the analysis, we have found that in case of stationary convection, the magnetic field has stabilizing effect on the system for $\lambda > 0$ and has a destabilizing effect for $\lambda < 0$. For stationary convection, it is also found that suspended particles and medium permeability have destabilizing effect on the system under the condition $\lambda > 0$ whereas for $\lambda < 0$, the nature of their effect reverses i.e. both parameters stabilizes the system for $\lambda < 0$. Further, the case of oscillatory mode is also considered. It is found that the principle of exchange of stabilities is valid for the problem under certain condition. The effect of all studied parameters on ferromagnetic fluid is also verified numerically.

Introduction:-

Ferromagnetic fluid (also called ferrofluid or magnetic fluid) is electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon or organic solvent etc. These colloidal particles are coated with a stabilizing dispersing agent (surfactants) who prevents particle agglomeration even when a strong magnetic field gradient is applied to the ferromagnetic fluid. These suspensions are stable and maintain their properties at extreme temperatures and over a long period of time. A detailed account on the stability of ferromagnetic fluid has been given by Rosenweig (1985) in his monograph. This monograph reviews several applications of heat transfer through ferromagnetic fluid. Ferromagnetic fluids have very large potential applications in electronic devices, mechanical engineering, material science, analytical instrumentation, medicines, optics, arts etc. Owing the applications of the ferromagnetic fluid, its study is important to researchers. Ferrofluid technology is well established and capable of solving a wide variety of technical problems. There are many successful applications of this engineering material and there is an immense scope of further research. There are various stability problems on ferromagnetic fluids. Many investigators (Siddheswar, 1993, 95, 2003; Aniss, et al., 1993, 2001 and Sunil, et al., 2004) have been considered the Bénard convection in ferromagnetic fluids. In all the above studies, the ferromagnetic fluid has been considered to be clean. In many situations the fluid is not pure but contains suspended particles. In 1962, Saffman considered the stability of laminar flow of a dusty gas. The effect of suspended particles on the onset of Bénard convection has been considered by Scanlon and Segel (1973), where as Sharma, et al., (1976) have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. They found that the critical Rayleigh number is reduced because of the capacity of the particles.

The effect of dust particles on ferrofluids heated and soluted from below is investigated by Sunil el al., 2006. Aggarwal et al. (2012) studied the effect of suspended particles, magnetic field and rotation on the thermal stability of ferromagnetic fluid and found that suspended particles have destabilizing effect whereas the magnetic field and magnetization have stabilizing effect on the system under certain conditions. Effects of magnetic field and suspended particles on ferrofluid have been studied by many authors (Scanlon and Segel, 1973; Sharma et al., 1976; Sunil et al., 2004; Sunil et al., 2005; Aggarwal and Prakash, 2009) but they all have assumed the constant gravity field. However, the earth's gravity varies with height from its surface. But usually we neglect this variation of gravity for laboratory purposes and treat the field as a constant. This may not be the case for large scale flows in the ocean or the atmosphere. Considering the gravity as a quantity varying with distance from the centre can become imperative. Pradhan and Samal (1987) have studied the thermal stability of a fluid layer under gravitational field. The instability of streaming the Rivlin-Ericksen fluid in porous medium in hydromagnetics and the thermosolutal instability of the Rivlin-Ericksen fluid in the presence of magnetic field and variable gravity field in porous medium is studied by Sharma and Rana in 1999 and 2003. The stability of Rivlin-Ericksen elastic-viscous rotating fluid permited with suspended particles under variable gravity field in porous medium is studied by Rana and Kumar (2010).

In this article, we have studied the effect of suspended particles and magnetic field on thermal convection in ferromagnetic fluid with a varying gravity field saturated in a porous medium.

Mathematical Formulation of the Problem:-

Here we consider an infinite horizontal layer of ferromagnetic fluid of thickness 'd' bounded by the planes z = 0 and z = d in porous medium. The system is acted upon by a uniform magnetic field $\vec{\mathbf{H}}\,(0,0,\mathrm{H})$ and variable gravity field $\vec{\mathbf{g}}\,(0,0,-\mathrm{g})$, where $\mathrm{g} = \lambda \mathrm{g}_0$, g_0 is the value of g at z = 0 which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value g_0. The fluid layer is heated from below so that a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained across the layer (see Figure 1). The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε which is defined as the fraction of the total volume of the medium that is occupied by void space. Thus, the fraction $1-\varepsilon$ is occupied by solid.

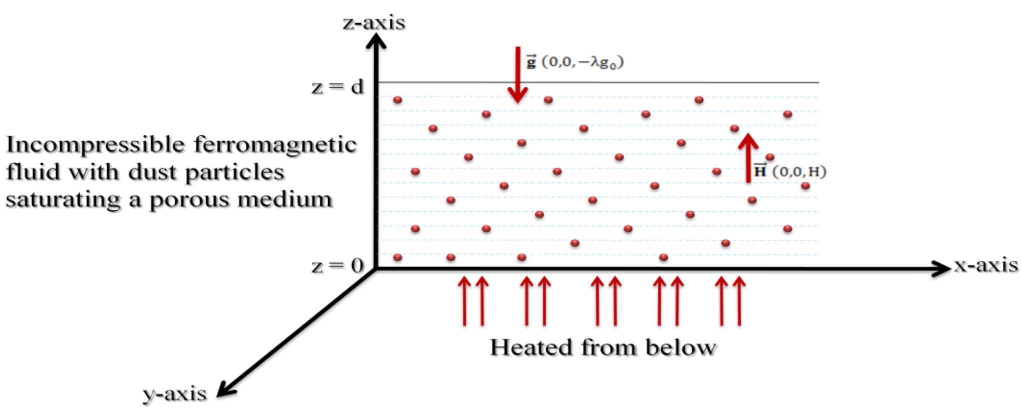

Fig 1: Geometrical Configuration

The mathematical equations governing the motion of ferromagnetic fluid under the Boussinesq approximation, saturating a porous medium for the above model are as follows:

The equation of continuity, conservation of momentum, temperature and equation of state of incompressible ferromagnetic fluid through porous medium are

$$\nabla.\vec{\mathbf{q}} = 0 \tag{1}$$

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{\mathbf{q}}}{\partial t} + \frac{1}{\varepsilon}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{q}}\right] = -\frac{1}{\rho_0}\nabla p + \frac{\rho\vec{\mathbf{g}}}{\rho_0} - \frac{1}{k_1}\nu\vec{\mathbf{q}} + \frac{M\nabla\vec{\mathbf{H}}}{\rho_0} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{\mathbf{H}})\times\vec{\mathbf{H}} + \frac{KN}{\rho_0\varepsilon}(\vec{\mathbf{q_d}} - \vec{\mathbf{q}}) \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\vec{\mathbf{q}}.\nabla)T+\frac{mN\,c_{pt}}{\rho_0 c_i}\left(\varepsilon\frac{\partial}{\partial t}+\overrightarrow{\mathbf{q_d}}\cdot\nabla\right)T=\kappa\nabla^2 T \quad (3)$$

$$\rho=\rho_0[1-\alpha(T-T_0)] \quad (4)$$

Equation of motion and continuity of dust particles are given by

$$mN\left[\frac{\partial\overrightarrow{\mathbf{q_d}}}{\partial t}+\frac{1}{\varepsilon}(\overrightarrow{\mathbf{q_d}}\cdot\nabla)\overrightarrow{\mathbf{q_d}}\right]=KN(\vec{\mathbf{q}}-\overrightarrow{\mathbf{q_d}}) \quad (5)$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot(N\overrightarrow{\mathbf{q_d}})=0 \quad (6)$$

Equation of magnetic field is given by

$$\varepsilon\frac{\partial\vec{\mathbf{H}}}{\partial t}=\nabla\times\left(\vec{\mathbf{q}}\times\vec{\mathbf{H}}\right)+\varepsilon\eta\nabla^2\vec{\mathbf{H}} \quad (7)$$

Where, $\vec{\mathbf{q}}(u,v,w)=$ filter velocity of pure fluid, $\overrightarrow{\mathbf{q_d}}(l,r,s)=$ velocity of suspended particles, $p=$ the fluid pressure, $\rho=$ fluid density, $N(\bar{x},t)=$ number density of suspended particles, $\bar{x}=(x,y,z)$, $\rho_0=$ reference density, $T_0=$ reference temperature, $K=6\pi\rho\nu\eta$ where η is the particle radius, is the stoke's drag coefficient, $T=$ temperature, $g=$ gravitational acceleration, $\alpha=$ thermal coefficient of expansion, $\varepsilon=$ medium porosity, $\mu_e=$ magnetic permeability, $\nu=$ kinematic viscosity of fluid, $k_1=$ medium permeability, $\kappa=\frac{\chi_T}{\rho_0 c_i}=$ thermal diffusivity, $E=\varepsilon+(1-\varepsilon)\frac{\rho_s}{\rho_0}\frac{c_s}{c_i}$, $\rho_s, c_s=$ density and specific heat of solid (porous matrix) material, $\rho_0, c_i=$ density and specific heat of fluid, $c_{pt}=$ specific heat of dust particles, $\chi_T=$ thermal conductivity, mN= mass of the particles per unit volume.

Assuming the fluid is electrically non-conducting and that the displacement current is negligible, Maxwell's equations becomes

$$\nabla.\vec{\mathbf{B}}=0 \quad \text{and} \quad \nabla\times\vec{\mathbf{H}}=0 \quad (8)$$

In Chu formulation of electrohydrodynamics, the relation between the magnetic field, magnetization and magnetic induction is

$$\vec{\mathbf{B}}=\mu_0\left(\vec{\mathbf{H}}+\vec{\mathbf{M}}\right) \quad (9)$$

Here, $\vec{\mathbf{M}}$ stands for magnetization, $\vec{\mathbf{H}}$ stands for the magnetic field intensity and $\vec{\mathbf{B}}$ for magnetic induction.

Assuming magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field and temperature, so that

$$\vec{\mathbf{M}}=\frac{\vec{\mathbf{H}}}{H}M\left(\vec{\mathbf{H}},\vec{\mathbf{T}}\right) \quad (10)$$

Where, $\vec{\mathbf{H}}=(0,0,H)$, i.e. $\vec{\mathbf{H}}=H\widehat{\mathbf{e}_z}$, $\widehat{\mathbf{e}_z}$ is the unit vector along z-axis and H is the uniform magnetic field of the fluid layer and

$$H=\left|\vec{\mathbf{H}}\right|, \quad M=\left|\vec{\mathbf{M}}\right| \quad \text{and} \quad B=\left|\vec{\mathbf{B}}\right|$$

Generally, for completing a system, it is necessary that the equation of state will specify M in two thermodynamics variables (say H and T), but in present study, we consider that the magnetization is independent of the magnetic field intensity i.e. $M=M(T)$. Thus, as a first approximation, we assume that

$$M=M_0[1-\gamma(T-T_0)] \quad (11)$$

Where M_0 is the magnetization at $T=T_0$ and $\gamma=\frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_H$

The basic state is assumed to be quiescent state and is given by

$$\vec{\mathbf{q}}=\vec{\mathbf{q}}_b=(0,0,0),\ \overrightarrow{\mathbf{q_d}}=(\overrightarrow{\mathbf{q_d}})_b=(0,0,0),\ p=p_b(z),$$
$$\vec{\mathbf{H}}=\vec{\mathbf{H}}_b(z),\ \vec{\mathbf{B}}=\vec{\mathbf{B}}_b(z),\ N=N_b=N_0(\text{Constant}),\ T=T_b(z)=-\beta z+T_0$$
$$\rho=\rho_b=\rho_0(1+\alpha\beta z),\ M=M_0(1+\gamma\beta z) \quad (12)$$

The Perturbations Equations

Let $\vec{\mathbf{q}}'(u,v,w)$, $\overrightarrow{\mathbf{q_d}'}$, p', ρ', M', θ $N'$and $\vec{\mathbf{H}}'\left(H'_x, H'_y, H'_z\right)$ denote respectively the small perturbations in fluid velocity, dust particles velocity, pressure, density, magnetization, temperature, number density of suspended particles and magnetic field. So that

$$\vec{\mathbf{q}}=\vec{\mathbf{q}}_b+\vec{\mathbf{q}}',\ \overrightarrow{\mathbf{q_d}}=(\overrightarrow{\mathbf{q_d}})_b+\overrightarrow{\mathbf{q_d}'},\ p=p_b+p',\ M=M_b+M',\ \vec{\mathbf{H}}=\vec{\mathbf{H}}_b+\vec{\mathbf{H}}',\ T=T_b+\theta,\ N=N_b+N',\ \rho=\rho_b+\rho'$$

Applying these perturbations and linearising equations (1) – (11), we get

$$\nabla.\vec{\mathbf{q}}'=0 \quad (13)$$

$$\frac{1}{\varepsilon}\frac{\partial\vec{\mathbf{q}}'}{\partial t}=-\frac{1}{\rho_0}\nabla p'-\frac{\lambda g_0\rho'}{\rho_0}\widehat{\mathbf{e}_z}-\frac{KN_0}{\rho_0\varepsilon}\left(\vec{\mathbf{q}}'-\overrightarrow{\mathbf{q_d}}'\right)-\frac{1}{k_1}\nu\vec{\mathbf{q}}'+\frac{M'\nabla\vec{\mathbf{H}}}{\rho_0}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{\mathbf{H}}'\right)\times\vec{\mathbf{H}} \quad (14)$$

$$(E+h\varepsilon)\frac{\partial\theta}{\partial t}=\beta(w+hs)+\kappa\nabla^2\theta \tag{15}$$

where $h=\frac{mN_0c_{pt}}{\rho_0c_i}$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\overrightarrow{\mathbf{q_d}}'=\vec{\mathbf{q}}'\Rightarrow\overrightarrow{\mathbf{q_d}}'=\frac{\vec{\mathbf{q}}'}{L_0},\ \text{where } L_0=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right) \tag{16}$$

$$\varepsilon\frac{\partial N'}{\partial t}+N_0\left(\nabla\cdot\overrightarrow{\mathbf{q_d}}'\right)=0 \tag{17}$$

$$\varepsilon\frac{\partial\vec{\mathbf{H}}'}{\partial t}=\nabla\times\left(\vec{\mathbf{q}}'\times\vec{\mathbf{H}}\right)+\varepsilon\eta\nabla^2\vec{\mathbf{H}}' \tag{18}$$

$$\rho'=-\rho_0\alpha\theta \tag{19}$$

$$M'=-M_0\gamma\theta \tag{20}$$

Now, eliminating $\overrightarrow{\mathbf{q_d}}'$ in equation (14) and (15) with the help of equation (16), we get

$$\frac{L_0}{\varepsilon}\frac{\partial\vec{\mathbf{q}}'}{\partial t}=L_0\left[-\frac{1}{\rho_0}\nabla p'+\lambda g_0\alpha\theta\widehat{\mathbf{e}_z}-\frac{mN_0}{\rho_0\varepsilon L_0}\frac{\partial\vec{\mathbf{q}}'}{\partial t}-\frac{1}{k_1}\nu\vec{\mathbf{q}}'-\left(\frac{M_0\gamma\nabla H}{\rho_0}\right)\theta\widehat{\mathbf{e}_z}+\frac{\mu_eH}{4\pi\rho_0}\left(\nabla\times\vec{\mathbf{H}}'\right)\times\widehat{\mathbf{e}_z}\right] \tag{21}$$

$$L_0\left[(E+h\varepsilon)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\beta(h+L_0)w \tag{22}$$

Writing the scalar components of equation (21) and eliminating $\nabla p'$, u, v, H'_x, H'_y between them by using equation (13), we get

$$\left[\frac{1}{\varepsilon}\left(1+\frac{M}{\tau\frac{\partial}{\partial t}+1}\right)\frac{\partial}{\partial t}+\frac{\nu}{k_1}\right]\nabla^2 w=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\lambda g_0\alpha-\frac{M_0\gamma\nabla H}{\rho_0}\right)\theta+\frac{\mu_eH}{4\pi\rho_0}\nabla^2\frac{\partial}{\partial z}H'_z \tag{23}$$

Taking the z- component of equation (18), we get

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)H'_z=\frac{H}{\varepsilon}\frac{\partial w}{\partial z} \tag{24}$$

From equation (15), we obtain

$$\left[(E+h\varepsilon)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\beta\left(1+\frac{h}{\tau\frac{\partial}{\partial t}+1}\right)w \tag{25}$$

Where $M=\frac{mN_0}{\rho_0}$ and $\tau=\frac{m}{K}$

Normal Mode Analysis

Now we analyze the perturbations into normal modes by assuming the following forms of perturbation quantities

$$[w,\theta,H'_z]=[W(z),\Theta(z),Z(z)]e^{(ik_xx+ik_yy+nt)} \tag{26}$$

Where k_x and k_y are wave numbers along x and y directions respectively, $a=\sqrt{k_x{}^2+k_y{}^2}$ is the resultant wave number of the disturbance and n is the growth rate (Complex constant). For functions with this dependence on x, y and t, $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=-a^2$, $\nabla^2=\frac{\partial^2}{\partial z^2}-a^2$

Using equation (26), equations (23) – (25) in non-dimensional form becomes

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\right](D^2-a^2)W=\frac{-\alpha a^2d^2}{\nu}\left(\lambda g_0-\frac{M_0\gamma\nabla H}{\rho_0\alpha}\right)\Theta+\frac{\mu_eHd}{4\pi\rho_0\nu}D(D^2-a^2)Z \tag{27}$$

$$(D^2-a^2-\sigma p_2)Z=-\frac{Hd}{\varepsilon\eta}DW \tag{28}$$

$$(D^2-a^2-E_1\sigma p_1)\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{H_1+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{29}$$

Where $H_1=1+h$ and we expressed in non-dimensional form by using the following non-dimensional parameters $a=\frac{a^*}{d}$, $\sigma=\frac{nd^2}{\nu}$, $D^*=dD$, $p_1=\frac{\nu}{\kappa}$ is the prandtl number, $p_2=\frac{\nu}{\eta}$ is the magnetic prandtl number, $P_l=\frac{k_1}{d^2}$, $E_1=E+h\varepsilon$, $\tau_1=\frac{\tau\nu}{d^2}$ (dropping $*$ for convenience)

Exact Solution for Free Boundaries

Here, we have considered that both the boundaries are free and perfect conductor of heat. The boundary conditions for the problem are (Chandrasekhar, 1981)

$$W=D^2W=0,\ \Theta=DZ=0 \text{ when } Z=0 \text{ and } 1 \tag{30}$$

Eliminating Θ and Z from (27), (28) and (29), we get-

$$\frac{a^2\lambda R_f}{(D^2-a^2-E_1\sigma p_1)}\left(\frac{H_1+\tau_1\sigma}{1+\tau_1\sigma}\right)W=\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\right](D^2-a^2)W+\frac{Q}{\varepsilon}\frac{D^2(D^2-a^2)}{(D^2-a^2-\sigma p_2)}W \quad (31)$$

Where $R_f=\left(g_0-\frac{M_0\gamma\nabla H}{\lambda\rho_0\alpha}\right)\frac{\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number for ferromagnetic fluids with varying gravity field. If $\lambda=1$, then this reduces to general Rayleigh number (Aggarwal and Makhija, 2012). $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\, \eta}$ is the Chandrasekhar number.

If $\lambda>0$, $g_0>\frac{M_0\gamma\nabla H}{\lambda\rho_0\alpha}$, then $R_f<R$, this implies that the convection starts in the ferrofluid at a higher thermal Rayleigh number.

If $\lambda<0$, then $R_f>R$, which implies that the convection starts in the ferrofluid at a lower thermal Rayleigh number.

Hence the proper solution for W characterizing the lowest mode is

$$W=W_0\sin\pi z \quad (32)$$

Where, W_0 is a constant. Substituting the proper solution (32) in equation (31), we get

$$x\lambda R_1=\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\tau_1\sigma_1\pi^2}\right)+\frac{1}{P}\right]\left(\frac{1+i\tau_1\sigma_1\pi^2}{H_1+i\tau_1\sigma_1\pi^2}\right)(1+x+iE_1\sigma_1p_1)(1+x)+\frac{Q_1}{\varepsilon}\left(\frac{1+i\tau_1\sigma_1\pi^2}{H_1+i\tau_1\sigma_1\pi^2}\right)\frac{(1+x+iE_1\sigma_1p_1)}{(1+x+i\sigma_1p_2)}(1+x) \quad (33)$$

Where $R_1=\frac{R_f}{\pi^4}$, $Q_1=\frac{Q}{\pi^2}$, $x=\frac{a^2}{\pi^2}$, $i\sigma_1=\frac{\sigma}{\pi^2}$, $P=\pi^2 P_l$

Equation (33) is the required dispersion relation including the effect of magnetic field, medium permeability, dust particles, kinematic viscosity and variable gravity field on the thermal convection of ferromagnetic fluid in porous medium. This relation agrees with the dispersion relation derived by Makhija (2012) for Rivlin-Ericksen fluid, if rotation and solute concentration is removed from his study.

The Case of Stationary Convection

For the case of stationary convection, the marginal state will be characterized by $\sigma_1=0$, therefore the dispersion relation (33) reduces to

$$R_1=\frac{(1+x)}{x\lambda H_1}\left[\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right] \quad (34)$$

The above equation expresses the modified Rayleigh number R_1 as a function of modified magnetic field parameter Q_1, suspended particles parameter H_1, medium permeability parameter P and dimensionless wave number x.

To study the effect of magnetic field, suspended particles and medium permeability, we examine the nature of $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dH_1}$ and $\frac{dR_1}{dP}$ analytically.

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{x\lambda H_1\varepsilon} \quad (35)$$

This equations shows that magnetic field Q_1 has stabilizing effect when $\lambda>0$ while it has destabilizing effect when $\lambda<0$.

$$\frac{dR_1}{dH_1}=-\frac{(1+x)}{x\lambda H_1^2}\left[\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right] \quad (36)$$

This is negative. This shows that the effect of suspended particles is to destabilize the system when $\lambda>0$ and to stabilize the system when $\lambda<0$.

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{x\lambda H_1P^2} \quad (37)$$

This shows that the medium permeability has a destabilizing effect for $\lambda>0$ and stabilizing effect on the system for $\lambda<0$.

The dispersion relation (34) is analyzed numerically also. In Figure 2, R_1 is plotted against modified magnetic field parameter Q_1 for $H_1=10$, $P=0.13$, $\varepsilon=0.15$, $\lambda=2$, $x=2,4,6$ and in Figure 3, R_1 is plotted against wave number x for $H_1=10$, $P=0.2$, $\varepsilon=0.15$, $\lambda=2$, $Q_1=20,40,60$. Both the figures shows the stabilizing effect of magnetic field (for $\lambda>0$) as Rayleigh number increases with the increase in magnetic field parameter. In figure 4, R_1 is plotted against modified suspended particle parameter H_1 for $Q_1=30$, $P=0.13$, $\varepsilon=0.15$, $\lambda=2$, $x=1,8,15$ and in figure 5, R_1 is plotted against wave number x for $Q_1=30$, $P=0.2$, $\varepsilon=0.15$, $\lambda=2$, $H_1=10,30,50$. Both the figures shows the destabilizing effect of suspended particles as the Rayleigh number decreases with the increase in suspended particles parameter H_1 for the case $\lambda > 0$. Figure 6 shows the variation of R_1 with medium permeability parameter P for $H_1 = 10$, $Q_1 = 30$, $\varepsilon = 0.15$, $\lambda = 2$, $x = 1, 8, 15$ and Figure 7 shows the variation of R_1 with wave number for $H_1 = 10$, $Q_1 = 30$, $\varepsilon = 0.15$, $\lambda = 2$, $P = 0.01, 0.05, 0.09$. In both the figures, Rayleigh number decreases with the increase in medium permeability parameter P which confirms the destabilizing effect of medium permeability on the system for the case $\lambda > 0$.

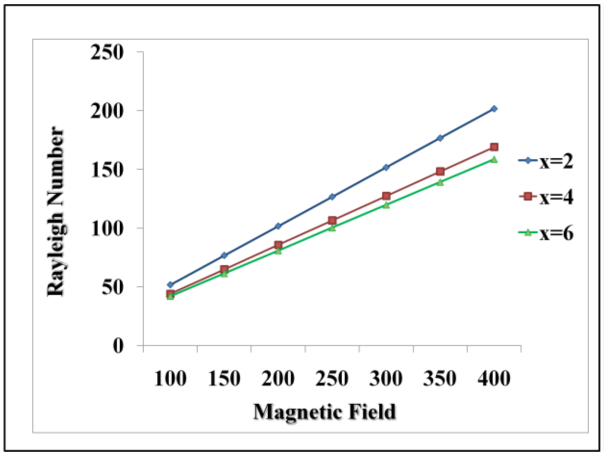

Fig 2: Variation of R_1 with Q_1

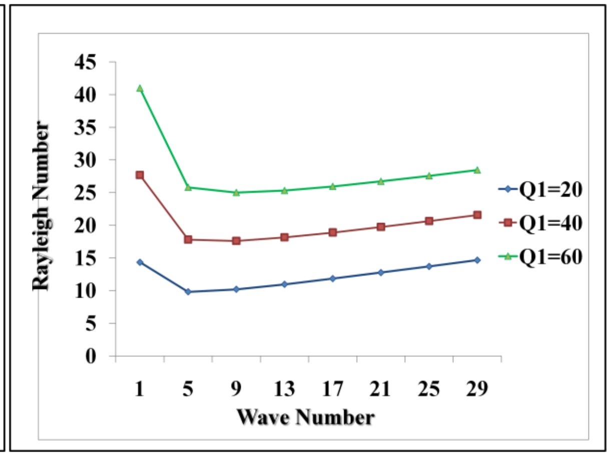

Fig 3: Variation of R_1 with x

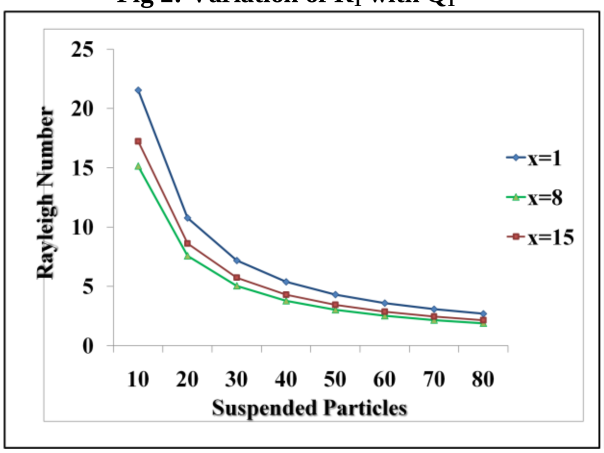

Fig 4: Variation of R_1 with H_1

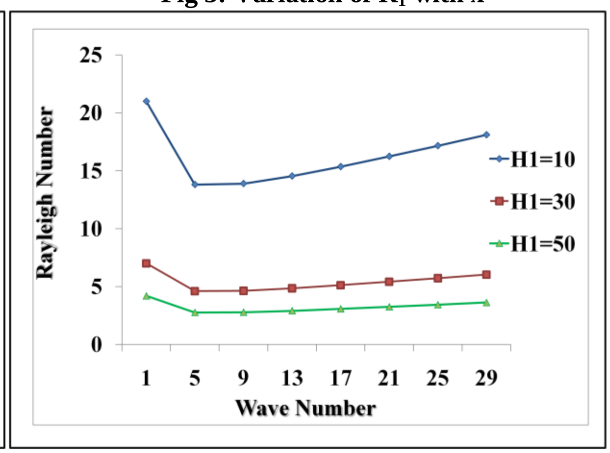

Fig 5: Variation of R_1 with x

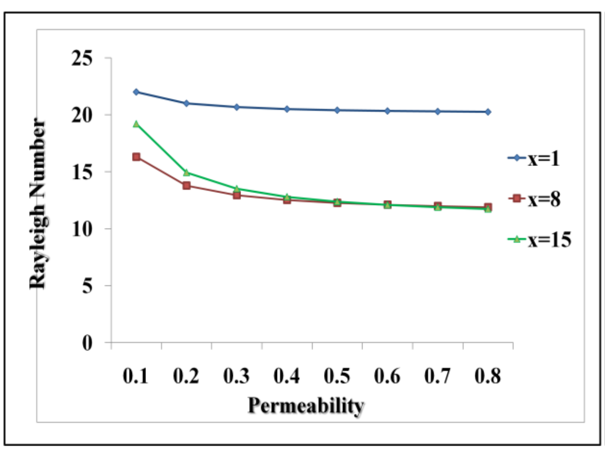

Fig 6: Variation of R_1 with P

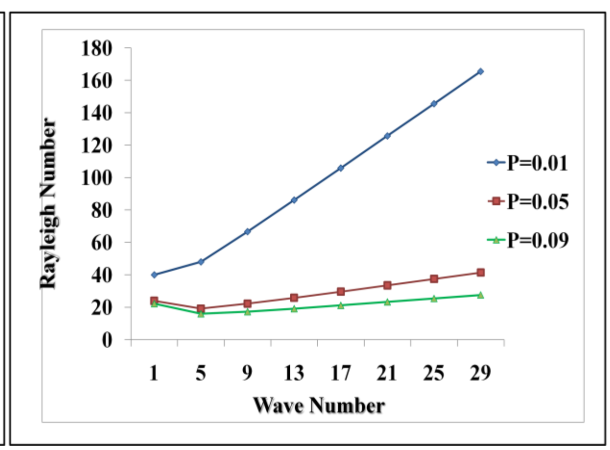

Fig 7: Variation of R_1 with x

The Case of Oscillatory Mode

Multiplying equation (27) by W^* (complex conjugate of W) and integrating over the range of z, we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\right]\int_0^1 W^*(D^2-a^2)W\,dz+\frac{\alpha a^2 d^2}{\nu}\left(\lambda g_0-\frac{M_0\gamma\nabla H}{\rho_0\alpha}\right)\int_0^1 W^*\Theta\,dz-\frac{\mu_e Hd}{4\pi\rho_0\nu}\int_0^1 W^*(D^2-a^2)DZ\,dz=0 \tag{38}$$

Integrating equation (38), using boundary conditions together with (28) and (29), we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\right]I_1-\frac{\alpha a^2\kappa}{\nu\beta}\left(\lambda g_0-\frac{M_0\gamma\nabla H}{\rho_0\alpha}\right)\left(\frac{1+\tau_1\sigma^*}{H_1+\tau_1\sigma^*}\right)(I_2+E_1\sigma^* p_1 I_3)+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_4+\sigma^* p_2 I_5)=0 \tag{39}$$

Where $I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz$, $I_2=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz$, $I_3=\int_0^1|\Theta|^2\,dz$

$I_4=\int_0^1(|D^2Z|^2+2a^2|DZ|^2+a^4|Z|^2)\,dz$, $I_5=\int_0^1(|DZ|^2+a^2|Z|^2)\,dz$ (40)

and σ^* is the complex conjugate of σ. The integrals I_1 to I_5 all are positive definite. Putting $\sigma=i\sigma_i(\sigma^*=-i\sigma_i)$ in equation (39) and equating imaginary parts, we get

$$\sigma_i\left[\begin{array}{l}\frac{1}{\varepsilon}\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1+\frac{\alpha a^2\kappa}{\nu\beta}\left(\lambda g_0-\frac{M_0\gamma\nabla H}{\rho_0\alpha}\right)\left(\frac{H_1-1}{H_1^2+\tau_1^2\sigma_i^2}\right)\tau_1 I_2\\ \quad+\frac{\alpha a^2\kappa}{\nu\beta}\left(\lambda g_0-\frac{M_0\gamma\nabla H}{\rho_0\alpha}\right)\left(\frac{H_1+\tau_1^2\sigma_i^2}{H_1^2+\tau_1^2\sigma_i^2}\right)E_1\sigma_i p_1 I_3+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2 I_5\end{array}\right]=0 \tag{41}$$

If $\lambda g_0>\frac{M_0\gamma\nabla H}{\rho_0\alpha}$, then the terms in the bracket are positive definite which implies that $\sigma_i=0$. Therefore, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied if $\lambda>\frac{M_0\gamma\nabla H}{\rho_0\alpha g_0}$.

Conclusions:-

In this article, we have studied the effect of magnetic field, suspended particles, permeability and variable gravity field on thermal convection of ferromagnetic fluid saturating a porous medium. The conclusions from the analysis are as follows:

1) For stationary convection, magnetic field has stabilizing effect if $\lambda>0$, while it has destabilizing effect when $\lambda<0$.
2) Suspended particles has destabilizing effect if $\lambda>0$, while it has stabilizing effect when $\lambda<0$.
3) When gravity increases upward (i.e $\lambda>0$), the medium permeability has a destabilizing effect whereas it has stabilizing effect for $\lambda<0$.
4) The principle of exchange of stabilities is valid under certain conditions.

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