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RESEARCH ARTICLE

MIRAJ'S CUBO: A COMPREHENSIVE EXPLORATION OF RELATION OF CUBIC IDENTITIES WITH SQUARE NUMBERS

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Abstract

In this paper, I will introduce Miraj's Cubo, which is a pioneering mathematical identity that presents an interestingly new look at the sum of cubes. We all know the conventional way to express the sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$, which is a cornerstone of algebra, majorly used in polynomial factorization, equation solving, and mathematical proofs (Hardy & Wright, 1979). While this classical identity has been valuable in a lot of terms, its form has remained largely static, leaving room for alternative explorations. In this work, I reimagined the sum of cubes in an entirely new light by expressing it as a difference of squares: $a^3 + b^3 = (a(m+a))^2 - (a(m-a))^2$, where the parameter m, called Miraj's Change, plays an important role in the following transformation. It is defined as: $m = (1/4)(1+(b/a)^3)$ This parameter encodes the relationship between a and b, which changes with the change in their comparative values. The introduction of 'm' makes the new formula not only present an alternative way of expressing the sum of cubes but also allow for a more intimate relationship between the two terms. Implications of this discovery are immense. In this paper, I have shown a detailed derivation and validation of Miraj's Cubo about how this transformation takes advantage of the elegance and simplicity of the difference of squares to offer new pathways for algebraic manipulation and computation. I delve into the theoretical importance of 'm' and how it balances the contributions of a and b in cubic identities, with the potential to shed light on new algebraic relationships. It will find broad applications both theoretically and practically. Theoretically, the new tools for polynomial identity analysis given by Miraj's Cubo provide insights into their geometric and algebraic properties. On a practical level, it holds great promises for simplifying unwieldy computations-particularly in modular arithmetic and in algorithmic contexts where the representation of differences of squares has significant computational advantages. Besides its mathematical utility, Miraj's Cubo has pedagogical value, serving as a novel teaching tool for advanced algebra. The ability it gives to link cubic and quadratic identities provides a fresh avenue for students to deepen their understanding of algebraic relationships. In the final analysis, this paper not only presents a new mathematical identity but also redefines the way I approach and interpret the sum of cubes. This identity has

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potential applications in theoretical algebra, modular arithmetic, and computational algorithms where differences of squares offer efficiency. By bridging the cubic expressions with the simplicity of squares, Miraj's Cubo invites further exploration for its broader implications and potential extensions in higher-dimensional algebra and beyond.

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Introduction:-

The identity $a^3+b^3=(a+b)\ (a^2-ab+b^2)$ has served as a cornerstone in algebraic theory, underpinning polynomial analysis, factorization, and equation solving (Algebra.com, n.d.).

Genesis of Miraj's Cubo

The inspiration for Miraj's Cubo arose from Ramanujan's inquiries into cubic identities, particularly his work on expressing numbers as sums of cubes in different ways (Yogananda, 2012). Whereas, I wondered if cubic sums could be expressed in fundamentally different ways utilizing the difference of squares instead of traditional linear and quadratic factors. The result was an elegant transformation, offering new algebraic tools and computational strategies.

Derivation of Miraj's Cubo:

Expansion of the Proposed Formula

The formula for Miraj's Cubo is:

$$a^3 + b^3 = \left(a(m+a)\right)^2 - \left(a(m-a)\right)^2$$

Expanding the right-hand side using the difference of squares, I obtained:

$$a^3 + b^3 = \left(a(m+a)\right)^2 - \left(a(m-a)\right)^2$$

$= \left(a^2(m+a)^2\right) - \left(a^2(m-a)^2\right)$

$=a^2((m+a)^2 - (m-a)^2)$

The expression$(m+a)^2 - (m-a)^2$ simplifies to:

$$(m+a)^2 - (m-a)^2$$

$=\left(m+a+(m-a)\right)\left(m+a-(m+a)\right)$

$= (2m\cdot 2a) = 4ma$

Substituting back, I obtain:

$$a^3 + b^3 = a^2 4ma = 4ma^3$$

$$b^3 = 4ma^3 - a^3$$

$$b^3 = a^3(4m-1)$$

$$\left(\frac{b}{a}\right)^3 = 4m-1$$

$$\left(\frac{b}{a}\right)^3 + 1 = 4m$$

Hence, $m = \left(\frac{1}{4}\right)\left(1 + \left(\frac{b}{a}\right)^3\right)$

Incorporation of ‘m’

Substituting $m = \left(\frac{1}{4}\right)\left(1 + \left(\frac{b}{a}\right)^3\right)$, I’ll get:$4ma^3 = a^3\left(1 + \frac{b^3}{a^3}\right) = a^3 + b^3$which verifies the formula.

Theoretical Insights into Miraj's Change ‘m’

Definition of ‘m’

The parameter ‘m’ is defined as m= $m = \left(\frac{1}{4}\right)\left(1 + \left(\frac{b}{a}\right)^3\right)$, describing it as the sum of the cube of the ratio $\frac{b}{a}$with 1, multiplied by $\frac{1}{4}$. On the other hand, it encodes the relative proportions of a and b, serving as a bridge that transforms the sum of cubes into a difference of squares.

Behaviour of ‘m’

- When b=0: m=$\frac{1}{4}$, highlighting the dominance of a^3.
- When b=a: m=$\frac{1}{2}$, reflecting the symmetry in contributions from $a^3$and b^3.
- When b>>a: m increases, capturing the rising influence of b^3.
- This dynamic adaptability makes m an insightful parameter in analysing the relationship between the terms in cubic identities.

Applications of Miraj's Cubo

Algebraic Simplification

Miraj's Cubo allows cubic expressions to be rewritten as differences of squares, streamlining algebraic manipulations in polynomial factorizations and quadratic forms.

Computational Mathematics

The formula offers a computational advantage in scenarios, where working with squares is more efficient than handling cubic terms, such as in modular arithmetic or algorithmic implementations.

Examples and Validation:

Example 1: a=2, b=1

Substitute a=2, b=1;

$m = \frac{1}{4}\left(1 + \left(\frac{1}{2}\right)^3\right) = \frac{9}{32}$.

The formula becomes:

$$a^3 + b^3 = \left(2 \cdot \left(\frac{9}{32}\right) + 2\right)^2 - \left(2 \cdot \left(\left(\frac{9}{32}\right) - 2\right)\right)^2$$

Simplifying, the left-hand side$2^3 + 1^3 = 9$ matches the right-hand side, verifying the identity.

Example 2: a=3, b=−2

Substitute a=3, b=−2

$$m = \frac{1}{4}\left(1 + \left(-\frac{2}{3}\right)^3\right) = \frac{19}{108}$$

The formula becomes:

$$3^3 + (-2)^3 = \left(3\left(\left(\frac{19}{108}\right) + 3\right)\right)^2 - \left(3\left(\left(\frac{19}{108}\right) - 3\right)\right)^2$$

Simplifying, the left-hand side $3^3+(-2)^3=19$ matches the right-hand side, verifying the identity.

Conclusion:-

Miraj's Cubo redefines the sum of cubes, offering a novel perspective by expressing it as a difference of squares. The introduction of Miraj's Change (m) establishes a dynamic interplay between the terms a and b, unveiling deeper algebraic relationships. This formula is not only a mathematical curiosity but also a versatile tool with applications in theory, computation, and education, paving the way for further exploration into higher-dimensional algebra and polynomial identities.

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